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A Denoising-Decomposition Model Combining TV Minimisation and Fractional Derivatives

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Abstract. A new image denoising-decomposition model is proposed. It is based on the TV minimisation and fractional derivatives of various order. It is shown that the texture details are better described by fractional derivatives of order greater than 1, whereas the noise part by fractional derivative of order smaller than 1, and this effect is used in image denoising. The model is able to eliminate staircase effect and keeps the edge and texture information of the image. Various experiments confirm the efficiency of the model.

AMS subject classifications: 68U10, 58E50

Key words: Image denoising, total variation model, fractional derivatives, partial differential equations.

1. Introduction

It is well known that noise is always present during the image acquisition, transmission and storage, affecting the quality of the image. Therefore, very often the observed image should be processed before we will use it. The corresponding process, called image denoising, aims to reduce noise and improve the observed image.

Image denoising is one of the most important tools in image processing. It suppresses noise and provides more accurate information for subsequent processing. To restore an image, various variational methods have been used, including one of the most popular total variation (TV) image denoising model [18]. The TV model is a regularising criterion for solving inverse problems. This model uses the bounded variation space and requires an assumption that ideal image has a bounded total variation and allows jumps. Therefore, the TV model is quite efficient in regularisation of images without smoothing the edges. There are various numerical algorithms developed for the TV model — e.g. an iterative method [19], a primal-dual method [6, 7], projection methods [2, 5], the split Bregman method [12] and so on. On the other hand, the TV model has a number of deficiencies.

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The preservation of the image texture details is not satisfactory, and while working with smooth regions, it can consider noise as an edge producing the staircase effect.

One way to overcome the drawbacks of this model is to introduce the adaptivity. An adaptive variant of TV model has been recently proposed by Lenzen and Berger [14], who established existence results for a class of adaptive TV regularisers in continuous setting. Another way to improve the TV model is the use of high-order derivatives. Thus Lysaker *et al.* [15] presented a four-order partial differential equation (PDE) denoising model (LLT model) with a relatively fast evolution of the shock signal. This model efficiently overcomes the staircase effect, restores smooth regions and preserves the texture information but the sharp jumps can not be well retained.

Taking into account the deficiencies of the variational model with integer order derivatives, a fractional derivative variational denoising model has been developed by Chen *et al.* [8]. The Bioucas majorisation-minimisation algorithm was used to decompose complex fractional TV optimisation problems into a set of linear optimisation problems that can be solved by the conjugate gradient algorithm. Zhang *et al.* [20] considered an adaptive fractional-order multi-scale denoising model that uses the local variance measures and the wavelet based estimation of singularity. A new class of fractional-order anisotropic diffusion equations for noise removal was introduced by Bai and Feng [3], who used the discrete Fourier transform to construct an iterative scheme in the frequency domain. Various experimental results show that fractional-order variational models efficiently diminishes the staircase effect, preserve the image details and improve its quality. Moreover, Dong and Chen [21] discussed theoretical aspects of the total fractional order variation model and suitable approximation methods. They employed the fractional derivatives of different order in the regularisation term of the objective function and proposed a unified variational framework for noise removal — cf. Ref. [10].

We note that images usually consist of a mix of edges, texture regions and a noise. To obtain an optimal denoising effect, we can use different approach to different parts of the image. Here we develop a denoising model, which combines total variation with fractional derivatives. The texture and noise parts are, respectively, represented by the fractional derivatives of the order greater and smaller than 1. This model inherits the advantages of TV and fractional derivative models, while avoiding some of their deficiencies.

The rest of the paper is organised as follows. Section 2 introduces the TV model and fractional derivatives. Section 3 presents a new denoising model and a minimisation algorithm. In Section 4 we discuss the results of numerical experiments for real and synthetic images and compare various denoising models. A brief summary and conclusions are in Section 5.

2. Preliminaries

Let $u : \Omega \to R$ be an ideal undistorted image and $f : \Omega \to R$ the observed version of u. The TV model can be formulated as an optimisation problem — viz. one has to find the minimum