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A Fast Linearised Augmented Lagrangian Method for a Mean Curvature Based Model

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Abstract. A simple and efficient algorithm for solving a mean curvature based model is proposed. It uses linearisation technique and allows to find closed form solutions of all the subproblems involved. The experimental results show that the method is more efficient in terms of CPU time than the augmented Lagrangian methods considered earlier. Numerical examples demonstrate the convergence of the method.

AMS subject classifications: 65M55, 68U10, 94A08

Key words: Image denoising, mean curvature, linearised augmented Lagrangian method, closed form solution, shrinkage operator.

1. Introduction

Image denoising is an important technique to recover an original image u from the observed noisy image f. There are distinct approaches to this problem, including wavelet-based methods [7, 28], probability and statistics-based methods [4, 27] and variational methods [1, 13, 21]. The total variation (TV) model, also called the Rudin-Osher-Fatemi (ROF) model [20], deals with the Euler-Lagrange equation often solved by a time marching method [26]. Nevertheless, the convergence of such methods can be slow due to influence of the Courant-Friedrichs-Lewy (CFL) condition. Therefore, a number of fast algorithms have been recently developed to overcome the deficiencies of the ROF model and reduce computational cost. Thus Goldstein and Osher [10] proposed a split Bregman method,

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which solves the corresponding linear systems by Gauss-Seidel iterations. For periodic boundary conditions, the linear systems generated by the split Bregman method can be solved by fast Fourier transform (FFT) [30]. On the other hand, Tai and Wu [25] applied an augmented Lagrangian method, which turned out to be equivalent to the split Bregman method [29].

Avoiding the use of FFT and Gauss-Seidel iterations in the ROF, Duan and Huang [8] proposed a fixed-point augmented Lagrangian method for TV minimisation problems. Recently, a linearised technique has been successfully adopted in various optimisation problems — e.g. in multiplicative image denoising [5], Poisson image restoration [14], image segmentation, reconstruction, and compressive sensing [15,23,31,36]. These works stimulated the search of efficient numerical algorithms for the mean curvature (MC) image denoising model [37] that can be formulated as the minimisation problem

$$\min_u \left\{ \int_\Omega \left| \nabla \cdot \left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}} \right) \right| + \frac{\lambda}{2} \int_\Omega (u-f)^2 \right\}, \tag{1.1}$$

where the weighting parameter $\lambda > 0$ controls denoising and $\nabla \cdot \left(\nabla u/(\sqrt{1+|\nabla u|^2})\right)$ is the mean curvature of the image surface $\phi(x,y,z) = u(x,y) - z = 0$.

The MC model can efficiently handle the flaws of the ROF model. Although the ROF model preserves edges and small scale characteristics, it often creates staircase effects and diminishes the image contrast. To reduce the staircase effects, a number of high order models have been developed — cf. Refs. [3, 16–18, 32, 34, 35, 37]. Among them, let us specifically note the MC [37], Euler's elastica [17] and Gaussian curvature (GC) [3] models, which are non-convex so that the numerical algorithms converge to the local optimal solution. This property is not common to the majority of convex models. Other approaches reducing the staircase effects are considered in Refs. [2,11,12].

The traditional methods for solving Euler's elastica model are usually time-consuming due to the high non-linearity of the Euler-Lagrange equation [22]. The use of the augmented Lagrangian method [24] improves the situation but FFT still has required to deal with a number of subproblems. Therefore, Duan *et al.* [9] introduced a new variable to simplify a subproblem. The related Euler-Lagrange equations have been solved by a Gauss-Seidel method. In addition to the reduction of computational cost, this approach can be applied to computational domains with non-periodic boundaries. There are other improvements connected with the fast linearised augmented Lagrangian method [33].

Note that the Euler-Lagrange equation for the MC model (1.1) has order four. Therefore, it removes the high frequency noise components faster, reducing the staircase effects and providing better approximation to the original image. It also preserves the object edges during the noise elimination. More importantly, this model does not change image contrast and the object corners. As was mentioned in [39], for cleaned images the MC model preserves image contrast better than the ROF model. In addition, for synthetic images, this model handles corners better than the ROF and Euler's elastica models.

Thus the MC model possesses a number of valuable properties and fast iterative algorithms for its solution have important theoretical and practical applications. There are