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Modelling the Floating Ladder Track Response to a Moving Load by an Infinite Bernoulli-Euler Beam on Periodic Flexible Supports

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Abstract. An infinite Bernoulli-Euler beam (representing the "combined rail" consisting of the rail and longitudinal sleeper) mounted on periodic flexible point supports (representing the railpads) has already proven to be a suitable mathematical model for the floating ladder track (FLT), to define its natural vibrations and its forced response due to a moving load. Adopting deliberately conservative parameters for the existing FLT design, we present further results for the response to a steadily (uniformly) moving load when the periodic supports are assumed to be elastic, and then introduce the mass and viscous damping of the periodic supports. Typical support damping significantly moderates the resulting steady deflexion at any load speed, and in particular substantially reduces the magnitude of the resonant response at the critical speed. The linear mathematical analysis is then extended to include the inertia of the load that otherwise moves uniformly along the beam, generating overstability at supercritical speeds — i.e. at load speeds notably above the critical speed predicted for the resonant response when the load inertia is neglected. Neither the resonance nor the overstability should prevent the safe implementation of the FLT design in modern high speed rail systems.

AMS subject classifications: 93A30, 74H10, 74H15, 74H55, 44A30

Key words: Bernoulli-Euler beam, periodic flexible supports, moving load, support damping, load inertia, floating ladder track, Fourier-Laplace transform solution.

1. Introduction

Despite the ubiquity of cross-tie rail tracks nowadays, in Britain and North America there were *longitudinal tie* tracks in some of the earliest railway developments using wooden sleepers (including primitive constructions from tall trees laid end to end), and much later with longitudinal sleepers of steel and then reinforced concrete. In a

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Figure 1: FLT test installation at the RTRI (courtesy H. Wakui).

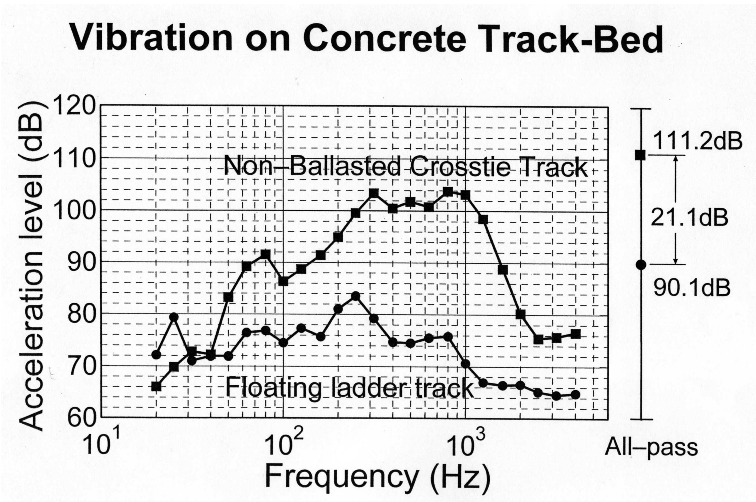

Figure 2: Vibration mitigation by the FLT (courtesy H. Wakui).

modern low maintenance version called a ladder track, the rails are fixed onto "ladder sleepers" consisting of two parallel longitudinal reinforced concrete beams separated by transverse connectors [1]. The *floating* ladder track (FLT), subsequently designed by H. Wakui and his colleagues at the Rail Technical Research Institute (RTRI) of Japan Railways, consists of a ladder track mounted upon discrete flexible supports (railpads) on a solid concrete track-bed — cf. Fig. 1. This FLT structure can significantly reduce traffic vibration and noise, as the experimental results in Fig. 2 reproduced from Ref. [2] demonstrated. The FLT has been installed in urban rail systems in the Tokyo region (e.g. as illustrated in Fig. 3), and in China there is now considerable interest in the potential for vibration mitigation by employing suitably based ladder tracks [3–5].

Although there is an extraordinarily large literature on modelling the dynamic behaviour of railway tracks, and in recent years the research effort has intensified with