East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.171012.061112a

Vol. **2**, No. 4, pp. 309-325
November 2012

Option Pricing of Weather Derivatives for Seoul

Jiwoon Kim[1], Dongwoo Sheen[1,2,*] and Sungwon Shin[2]

[1] *Department of Mathematics, Seoul National University, Seoul 151-747, Korea.*
[2] *Interdisciplinary Program in Computational Science & Technology, Seoul National University, Seoul 151-747, Korea.*

Received 17 October 2012; Accepted (in revised version) 6 November 2012

Available online 29 November 2012

Abstract. This article analyses temperature data for Seoul based on a well defined daily average temperature (DAT) derived from records dating from 1954 to 2009, and considers related weather derivatives using a previous methodology. The temperature data exhibit some quite distinctive features, compared to other cities that have been considered before. Thus Seoul has: (i) four clear seasons; (ii) a significant seasonal range, with high temperature and humidity in the summer but low temperature and very dry weather in winter; and (iii) cycles of three cold days and four warmer days in winter. Due to these characteristics, seasonal variance and oscillation in Seoul is more apparent in winter and less evident in summer than in the other cities. We construct a deterministic model for the average temperature and then simulate future weather patterns, before pricing various weather derivative options and calculating the market price of risk (MPR).

AMS subject classifications: 91G20, 60H15, 60J65

Key words: Weather derivatives, market price of risk, HDD, CDD, CAT.

1. Introduction

On January 4, 2010 there was a 25.8 centimeters snowfall in the central area of Korea encompassing the Capital Region and Gangwon-do, a record-breaking event since 1937. This heavy snowfall temporarily paralyzed transportation in that large area, and caused numerous accidents on the icy roads. Many agricultural facilities, including the ginseng greenhouses, were also broken by the weight of the piled-up snow. The loss of property caused by this snowfall was estimated to total 10.6 billion won. Apart from heavy snowfalls, the extreme weather events in Korea include unexpectedly intensive typhoons, heavy rains and heat waves in summer, and very cold winters. The cost of the annual average weather damage during the last ten years has been estimated to be more than 2 trillion

*Corresponding author. *Email addresses:* jiwoonkim83@gmail.com (J. Kim), sheen@snu.ac.kr (D. Sheen), swshin10@snu.ac.kr (S. Shin)

won, so financial losses due to weather risks should be covered by adequate weather-related insurances and derivatives. However, the Korean insurance market is rather stagnant, especially in regard to weather risks. According to the General Insurance Association of Korea, the number of weather-related contracts was thirty-six in 2002, twenty-seven in 2003, and forty-one in 2004. Although the market may be growing, it is restricted to contingency insurance where the insurance companies compensate the insured for damages that actually happen, and the proper estimation of total losses between the policyholders and the companies remains highly controversial.

The importance of weather risk has been recognized in most developed countries, where it is fast becoming customary to provide against uncertain climatic change. The typical provision includes the introduction of weather derivatives and associated Risk Management. An early weather transaction was executed by Aquila Energy, which structured a dual-commodity hedge for the Edison Company in 1996. Over-the-counter (OTC) weather derivatives have been traded since 1997, and at the Chicago Mercantile Exchange (CME) since the summer of 1999. In September 2003, the CME launched seasonality products for ten new cities, and then monthly for a list of twelve cities in the USA that was expanded to include five European cities. The CME now offers temperature products for twenty-four cities in the USA, six in Canada, eleven in Europe, three in Australia, and three elsewhere in the Asia-Pacific — cf. Tables 1 and 2). In addition to the increasing number of cities covered at the CME, the volume of weather derivative contracts issued has significantly increased — from 630,000 in 2005 to 798,000 in 2006, and to nearly 1,000,000 in 2007 [12, 23]. Although the volume did fall by about 16 % in 2008, that occurred during the onset of the current global financial crisis.

With the rapid growth of weather-related industries, relevant futures prices have been studied extensively [2–6,9,11,13,15–17,19–22,25–29]. Since weather derivatives are non-tradable, no-arbitrage models (such as the Black-Scholes model) are inapplicable to pricing weather options. In 2000, Dornier & Querel [13] used mean-reverting Itô diffusions based on a standard Brownian motion to model Chicago temperature data. Brody *et al.*! [9] proposed another dynamical model based on a fractional Brownian motion, and Alaton *et al.* [2] applied the Ornstein-Uhlenbeck process with a monthly variation to analyze the temperature at the Bromma Airport, Stockholm. Benth *et al.* [4, 5] generalized Dornier & Querel's approach by employing continuous autoregressive (CAR) models to analyze temperature data at Stockholm; and Härdle & Cabrera [16] also applied the CAR approach to Berlin temperature data, but they considered a nonzero market price of risk (MPR). To date no significant research for Korean weather derivatives and pricing has been reported, and a weather market has yet to be introduced. However, in order to keep pace with the growth of world-wide weather markets, the Financial Supervisory Commission of Korea now seems to favour the introduction and development of weather derivatives. The Korea Meteorological Administration (KMA) has also recently announced it intends to develop a weather index effective from 2012, to serve as one basic reference.

In this paper, we analyse the Seoul temperature data and then price related weather options, using the approach adopted in Refs. [2], [5] and [16]. The paper is organized as follows. In Section 2, we construct our Seoul temperature model based on observed data.

Table 1: Weather product: Temperature on CME (December, 2010).

	Product name	Region
U.S	U.S. Cooling (Monthly/Seasonal) U.S. Heating (Monthly/Seasonal) U.S. Weekly Weather	Atlanta, Chicago, Cincinnati New York, Dallas, Philadelphia Las Vegas, Boston, Houston, etc.
Canada	Canada *CAT* (Monthly/Seasonal) Canada Cooling (Monthly/Seasonal) Canada Heating (Monthly/Seasonal)	Calgary, Edmonton Montreal, Toronto Vancouver, Winnipeg
Europe	Europe *CAT* (Monthly/Seasonal) Europe Heating (Monthly/Seasonal)	London, Paris, Amsterdam, Berlin Stockholm, Essen, Barcelona, Rome, etc.
Australia	Australia Cooling (Monthly/Seasonal) Australia Heating (Monthly/Seasonal)	Bankstown, Sydney Brisbane Aero, Melbourne
Asia-Pacific	Asia-Pacific (Monthly/Seasonal)	Hiroshima, Osaka, Tokyo

Table 2: Weather product on CME (December, 2010).

Index	Product name	Region
Hurricanes	Hurricane Hurricane Seasonal Hurricane Seasonal Maximum	Gulf Coast, Florida, Southern Atlantic Coast Northern Atlantic Coast, Eastern U.S. Cat-In-A-Box, Florida Gold Coast
Frost	Frost (Monthly/Seasonal)	
Snowfall	Snowfall (Monthly/Seasonal)	Boston, New York Central Park, Chicago, etc.
Rainfall	Rainfall (Monthly/Seasonal)	Chicago O'Hare International Airport

Put and call options are then priced in Section 3, based on temperature derivatives. Finally, in Section 4 the market price of risk (MPR) is calculated, using the Korea Composite Stock Price Index (KOSPI).

2. Temperature Derivatives for Seoul

We investigate the temperature data for Seoul in a somewhat different way from previous analyses for other places. Firstly, most researchers [2,3,5,6,13,16] have defined the daily mean temperature as the average of the maximum and minimum temperatures for that day, but we adopt the following definition for the daily average temperature.

Definition 2.1 (Daily average temperature (*DAT*))**.** From the year 1997, the **daily average temperature** T_t is defined to be the average temperature of 8 observed temperature values at the 03, 06, 09, 12, 15, 18, 21, 24 hour times during the day t; and before 1997, T_t is defined as the average temperature of 4 observed values at the 03, 09, 15, 21 hour times during the day t.

We begin with the 20440 *DAT* data recorded for the 56 years from 1954.01.01 to 2009.12.31 at Seoul, Korea (Fig. 1) obtained from the KMA [1]. Leap-year day data are excluded. These data, which basically contain seasonal periodicity and an increment trend

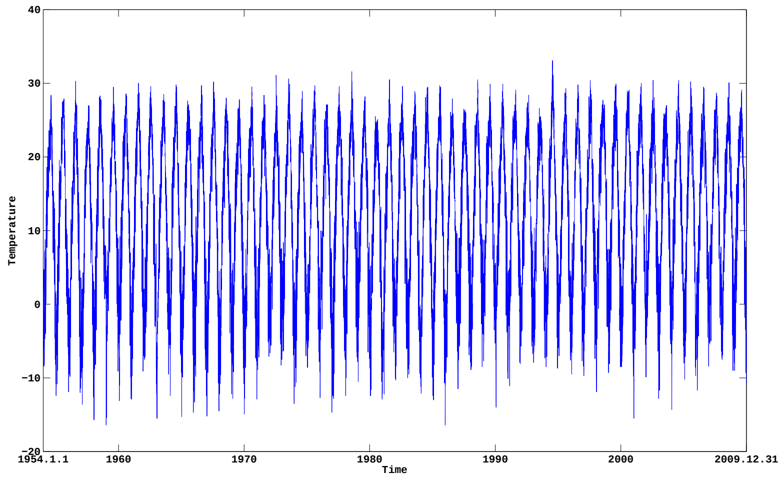

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Figure 1: Daily average temperature from 1954.01.01to 2009.12.31 at Seoul, Korea.

due to global warming, are to be interpreted as a function in time in the mathematical analysis. It seems natural to try to fit the yearly periodicity with a cosine polynomial and the global warming property with a linear term [2, 5, 16]. However, we assume

$$\Lambda_t = \lambda_0 + \lambda_1 t + \lambda_2 \cos \frac{2\pi(t-\lambda_3)}{365} + \lambda_4 \cos \frac{4\pi(t-\lambda_5)}{365}\,, \tag{2.1}$$

and remark that the difference between the form used by Benth *et al* [5] and Eq. (2.1) is the fourth term representing a half-year period. We include this term so that the ACF analysis in Section 3.2 works when seasonality in the squared residuals remains apparent, as will be seen in Fig. 7d. This seasonality is a distinct feature of the temperature at Seoul, compared to the other cities that have been considered elsewhere [2, 5, 7, 16].

Using the method of least squares, we get the coefficients $\lambda_0 = 11.1897$, $\lambda_1 = 0.0001$, $\lambda_2 = 13.9112$, $\lambda_3 = -161.2643$, $\lambda_4 = 1.3705$, and $\lambda_5 = -92.7957$. The fitted function and *DAT* are plotted in Fig. 2a. The blue line is the daily average temperature, while the red one is a fitted form using Eq. (2.1). Fig. 2b also depicts these data and fitted function, during the 10 years from 2000.1.1 to 2009.12.31. The coefficient λ_2 represents half of the temperature difference between the highest *DAT* in summer and the lowest *DAT* in winter, which is approximately $28°C$ — cf. Table 3. Compared to European cities (Berlin [16] and Stockholm [2]) and other Asian cities (Tokyo, Osaka, Taipei [7]), this value is much higher. The second term in Eq. (2.1) evidently reflects the greenhouse effect, with the annual average temperature rising as seen in Table 4.

Table 3: Temperature differences between the highest DAT in summer and the lowest DAT in winter.

Seoul	Berlin	Stockholm	Tokyo	Osaka	Taipei
$27.8°C$	$19.6°C$	$20.8°C$	$20.7°C$	$23°C$	$13.6°C$

Table 4: Monthly average temperatures during the 1950-1959 and 2000-2009 decades.

	1	2	3	4	5	6	7	8	9	10	11	12	Annual
1950's	-4.3	-1.0	3.5	10.6	16.4	20.6	23.9	25.1	20.1	13.0	6.6	-0.5	**11.2**
2000's	-1.6	1.0	6.0	12.8	18.3	22.5	24.9	25.7	21.6	15.3	7.5	0.4	**12.9**

3. Temperature Derivatives

There are three types of temperature indices used at the CME — viz. HDD, CDD and CAT. The HDD_n and CDD_n indices usually measure temperatures over a period starting from day u_1 to day u_n, with regard to heating and cooling when the DAT is below and above $18°C$, respectively. The CAT index accounts for the accumulated average temperature over day u_1, day $u_2, \cdots$, day u_n. Specifically, these three types are defined as follows:

$$HDD_n = \sum_{i=1}^{n} \max(18 - T_{u_i}, 0), \tag{3.1a}$$

$$CDD_n = \sum_{i=1}^{n} \max(T_{u_i} - 18, 0), \tag{3.1b}$$

$$CAT_n = \sum_{i=1}^{n} T_{u_i}. \tag{3.1c}$$

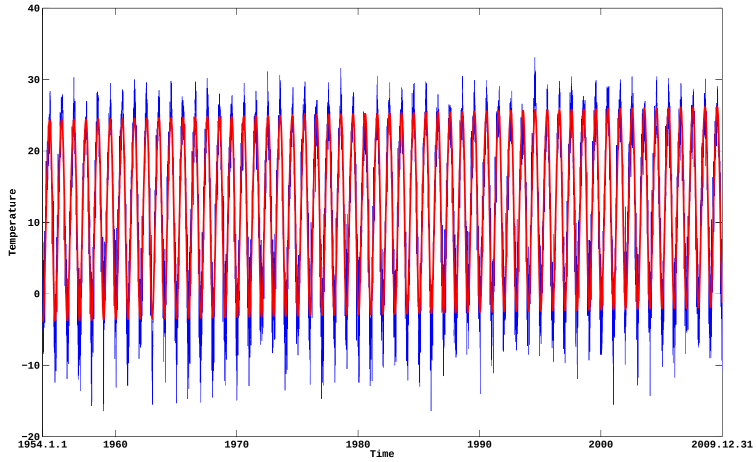
As shown in Table 1, the HDD and CDD indices are used in the USA, Canada and Australia. In Europe, the CAT index substitutes for the CDD index utilizing the HDD–CDD parity — thus

$$CDD_n - HDD_n = CAT_n - 18n. \tag{3.2}$$

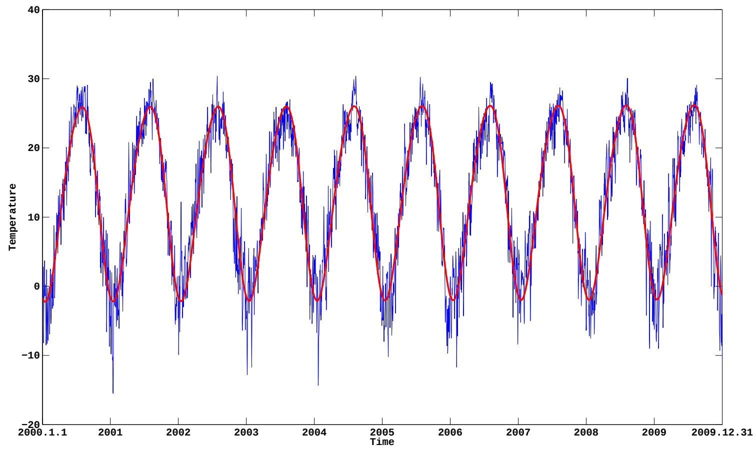
Similar to Japan, in Korea we may define the accumulated temperature index to be the sum over the period day u_1, day $u_2, \cdots$, day u_n of daily average temperatures, averaged over temperatures observed 8 times daily — i.e.

$$CAT_n = \sum_{i=1}^{n} T_{u_i}, \tag{3.3}$$

where $T_{u_i} = \sum_{j=1}^{8} \widetilde{T}_{u_i,j}/8$ involves the temperature $\widetilde{T}_{u_i,j}$ measured at hour $3j$ on day u_i, $j = 1, \cdots, 8$.



(a) 1954.01.01–2009.12.31



(b) 2000.01.01–2009.12.31

Figure 2: Seasonality effect and daily average temperatures for Seoul: the blue lines represent daily average temperatures (DAT), and the red lines fitted functions given by Eq. (2.1).

3.1. Option pricing for temperature derivatives 1: HDD and CDD

In order to calculate option pricing for the HDD and CDD, we follow the scheme of Alaton *et al.* [2] where the mean temperature T_t follows the Ornstein-Uhlenbeck process with mean reverting rate a — i.e.

$$dT_t = a(T_t^m - T_t)\,dt + \sigma_t\,dW_t\,, \tag{3.4}$$

Table 5: The quadratic variation $\widehat{\sigma}_\mu$ given by Eq. (3.7).

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
3.079	2.718	2.404	2.356	2.139	1.758	1.592	1.472	1.540	2.036	2.887	3.143

where T_t^m is the equilibrium or mean temperature value given by the expected temperature at day t from the past historical data for the temperature. We normally choose $T_t^m = \Lambda_t$, where Λ_t is given by Eq. (2.1). In Eq. (3.4), σ_t represents the degree of volatility around T_t^m, and W_t the Brownian motion on the probability space $(\Omega, \mathscr{F}, P)$ with a filtration $\{\mathscr{F}_t\}$.

To satisfy a mean-reverting property, we should add the term dT_t^m/dt to Eq. (3.4) — cf. Dornier & Querel [13]. We then arrive at the stochastic differential equation

$$dT_t = \left\{ \frac{dT_t^m}{dt} + a(T_t^m - T_t) \right\} dt + \sigma_t \, dW_t \tag{3.5}$$

with solution given by

$$T_t = (x - T_s^m)e^{-a(t-s)} + T_t^m + \int_s^t e^{-a(t-\tau)} \sigma_\tau \, dW_\tau \,, \tag{3.6}$$

where $x = T_s$ is the temperature observed at the starting day s. We need an estimation of both a and σ in Eq. (3.6). For $j = 1, \cdots, 365$, let $\overline{T}_j$ denote the average temperature of $DAT_{j+365(k-1)}$ where $k = 1, \cdots, 56$ — i.e. the average of all DAT's at the jth day of each year from 1954 to 2009. We then introduce an estimator σ based on the quadratic variation of T_t — i.e.

$$\widehat{\sigma}_\mu = \sqrt{\frac{1}{N_\mu} \sum_{j=0}^{N_\mu - 1} \left(\overline{T}_{j+1+s_\mu} - \overline{T}_{j+s_\mu} \right)^2} \,, \tag{3.7}$$

where μ denotes a specific month ($\mu = 1, \cdots, 12$) of the year and N_μ the number of days in that month, and s_μ indicates the number of days up to the last day of the previous month ($\mu - 1$).

Table 5 shows the quadratic variation $\widehat{\sigma}_\mu$ of each month, where it is notable that the variations in winter are about twice as large as those in the summer. As previously noted, there are cycles of three cold days and four warm days during winter, and the hot temperature in summer does not change significantly — and such characteristic features in the Korean peninsula should be taken into account in modelling relevant weather derivatives and option pricing.

With $\widehat{\sigma}_\mu$ from Table 5, we obtain the mean-reversion parameter value $\widehat{a}_n = 0.2748$, based on the martingale estimation functions method [8] where

$$\widehat{a}_n = \frac{\sum_{i=2}^n Y_{i-1} \left\{ T_i - T_{i-1} - dT_{i-1}^m/dt \right\}}{\sum_{i=2}^n Y_{i-1} \{ T_{i-1}^m - T_{i-1} \}} \,, \tag{3.8}$$

with $Y_{i-1} = (T^m_{i-1} - T_{i-1})/\overline{\sigma}^2_{i-1}$ $(i = 2, \cdots, n)$ and $\overline{\sigma}_j = \widehat{\sigma}_k$ if the j-th day starting from 1 January 1954 lies in the k-th month in some year.

In order to price call and put options for the HDD, we first compute the conditional expectation and variance. Let us consider option prices under a martingale measure Q characterized by the MPR θ, which is equivalent to P. From the Girsanov theorem, the expectation changes under the measure Q but the variance does not — i.e. we have

$$
\begin{aligned}
E^Q[T_{t_i}|\mathscr{F}_t] &= E^P[T_{t_i}|\mathscr{F}_t] - \frac{\theta\overline{\sigma}_i}{\widehat{a}_n}\left(1 - e^{-\widehat{a}_n(t_i - t)}\right) \\
&= (T_t - T^m_t)e^{-\widehat{a}_n(t_i - t)} + T^m_{t_i} - \frac{\theta\overline{\sigma}_i}{\widehat{a}_n}\left(1 - e^{-\widehat{a}_n(t_i - t)}\right), \qquad (3.9) \\
Var^Q[HDD_n|\mathscr{F}_t] &= \sum Var^Q[T_{t_i}|\mathscr{F}_t] + 2\sum_{i<j} Cov^Q[T_{t_i}, T_{t_j}|\mathscr{F}_t] \\
&= \sum \frac{\overline{\sigma}^2_i}{2\widehat{a}_n}\left(1 - e^{-\widehat{a}_n(t_i - t)}\right) + 2\sum_{i<j} e^{-\widehat{a}_n(t_j - t_i)}\frac{\overline{\sigma}^2_i}{2\widehat{a}_n}\left(1 - e^{-\widehat{a}_n(t_i - t)}\right),
\end{aligned}
$$

where Φ is the cumulative distribution function for the standard normal distribution. On setting

$$
\beta_n = (K - \mu_n)/\sigma_n\,, \quad \mu_n = E^Q[HDD_n|\mathscr{F}_t] = 18n - \sum E^Q[T_{t_i}|\mathscr{F}_t] \qquad (3.10)
$$

where $\sigma^2_n = Var^Q[HDD_n|\mathscr{F}_t]$, we find the price of the HDD call option given by

$$
\begin{aligned}
HDD_{call}(t) &= \exp[-r(t_n - t)]\, E^Q\Big[\max(HDD_n - K, 0)|\mathscr{F}_t\Big] \\
&= \exp[-r(t_n - t)]\left((\mu_n - K)\Phi(-\beta_n) + \frac{\sigma_n}{\sqrt{2\pi}}\exp\left(-\frac{\beta^2_n}{2}\right)\right). \qquad (3.11)
\end{aligned}
$$

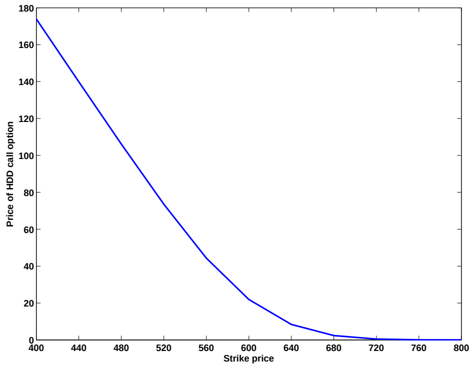
Further, the price of the HDD put option is likewise given by

$$
\begin{aligned}
HDD_{put}(t) &= \exp[-r(t_n - t)]\, E^Q[\max(K - HDD_n, 0)|\mathscr{F}_t] \\
&= \exp[-r(t_n - t)]\left[(K - \mu_n)\left(\Phi\left(\frac{K - \mu_n}{\sigma_n}\right) - \Phi\left(-\frac{\mu_n}{\sigma_n}\right)\right)\right. \\
&\qquad \left. + \frac{\sigma_n}{\sqrt{2\pi}}\left(\exp\left(-\frac{\beta^2_n}{2}\right) - \exp\left(-\frac{\mu^2_n}{2\sigma^2_n}\right)\right)\right]. \qquad (3.12)
\end{aligned}
$$

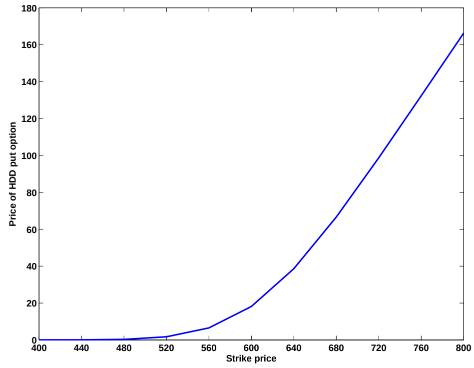
The formulae for CDD call and put options can be derived analogously, and are quite similar to Eqs. (3.11) and (3.12). It is notable that θ in Eq. (3.9) represents the market price of risk (MPR), discussed in detail in Section 4. From these equations, we get the option prices shown in Table 6, assuming that $MPR = 0$. The HDD and CDD call and put option prices with $r = 0.036$ are illustrated in Fig. 3 for January and August of 2011, respectively.

Table 6: Option prices: Market Price of Risk (MPR)=0, $r = 0.036$, and the trading date is the first of December for the HDD and the first of July for the CDD, respectively.

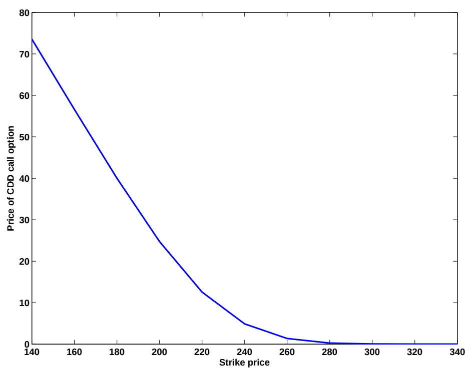
Index	HDD call	HDD put	CDD call	CDD put
Strike price	600	600	220	220
Measurement Period	Jan.2011	Jan.2011	Aug.2011	Aug.2011
Price	23.25	16.25	9.97	8.75



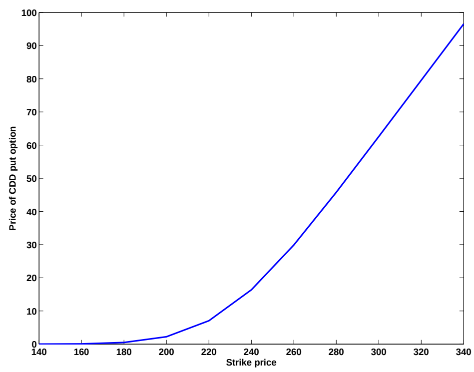
(a) HDD call



(b) HDD put



(c) CDD call



(d) CDD put

Figure 3: Option prices: Market Price of Risk (MPR) = 0, $r = 0.036$, for the HDD and CDD call and put options calculated for the months of January and August 2011, respectively. The measurement period is the entire month of August, and the trading date is the first of July in each case.

3.2. Option pricing for temperature derivatives 2: CAT

In this subsection, we estimate the CAT-futures price and its option value, using the Benth *et al.* [5] temperature dynamics model. Letting $\mathbf{W}_t$ denote the Brownian motion on the probability space $(\Omega, \mathscr{F}, P)$ with a filtration $\{\mathscr{F}_t\}_{0\le t\le \tau_{\max}}$, we now consider the vectorial Ornstein-Uhlenbeck process

$$d\mathbf{X}_t = A\mathbf{X}_t\,dt + \mathbf{e}_p\sigma_t\,d\mathbf{W}_t\,, \tag{3.13}$$

where $\mathbf{e}_p$ is the p-th unit vector in $\mathbb{R}^p$ and A is the $p\times p$ matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_p & -\alpha_{p-1} & -\alpha_{p-2} & \cdots & -\alpha_1 \end{bmatrix}.$$

Further, denoting by X_{qt} the q-th coordinate of the vector $\mathbf{X}_t$ we have

$$T_t = \Lambda_t + X_{1t}\,, \tag{3.14}$$

whence from Ito's lemma the temperature dynamic process is described as follows.

Lemma 3.1. *The stochastic process* $\mathbf{X}_t$ *in Eq.* (3.13) *can be expressed as*

$$\mathbf{X}_s = e^{A(s-t)}\mathbf{X}_t + \int_t^s e^{A(s-u)}\mathbf{e}_p\sigma_u\,d\mathbf{W}_u\,,$$

for $s \ge t \ge 0$.

We now proceed to consider the difference between the DAT and the seasonal behaviour

$$X_t = T_t - \Lambda_t\,. \tag{3.15}$$

The partial autocorrelation function (PACF) for X_t is plotted in Fig. 4, showing that the *AR*(3)-process [5] is suitable for the analysis of our data. The fitted autoregressive process using MATLAB corresponds to

$$X_{t+3} = 0.9385X_{t+2} - 0.3472X_{t+1} + 0.1132X_t + \sigma_t\epsilon_t\,, \tag{3.16}$$

where the seasonal variance σ_t^2 and the residual ϵ_t are computed as follows.

We first compute the residuals $\widehat{\epsilon_t} = X_{t+3} - 0.9385X_{t+2} + 0.3472X_{t+1} - 0.1132X_t$, as plotted in Figs. 5a and 5b together with their squares $\widehat{\epsilon_t}^2$. The ACF of the residuals and the squared residuals of AR(3) are plotted in Figs. 7a and 7c, showing that the residuals are close to zero but the squared residuals exhibit a high seasonality pattern. To avoid this problem, we consider the seasonal variance function σ_t^2 in Eq. (3.17). We also use the least squares method to get the parameters c_j's in the following formula:

$$\sigma_t^2 = c_1 + \sum_{j=1}^{4}\left(c_{2j}\cos\frac{2j\pi t}{365} + c_{2j+1}\sin\frac{2j\pi t}{365}\right)\,, \tag{3.17}$$

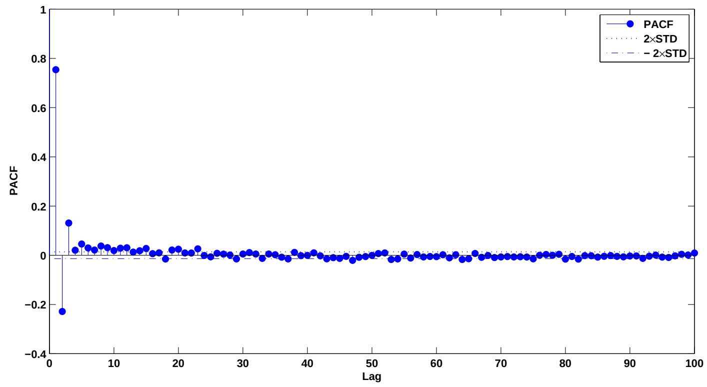
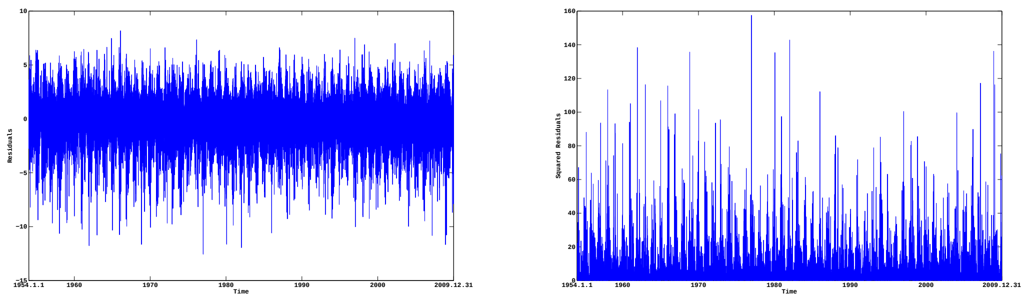


Figure 4: Partial autocorrelation function (PACF) for X_t during 1954.01.01 to 2009.12.31.



(a) Residuals (b) Squared residuals

Figure 5: Residuals $\widehat{\epsilon}_t$ and squared residuals $\widehat{\epsilon}_t^{\,2}$ for the AR(3) during 1954.01.01 to 2009.12.31.

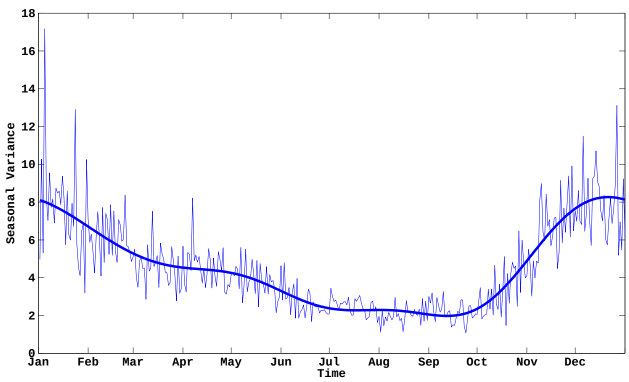
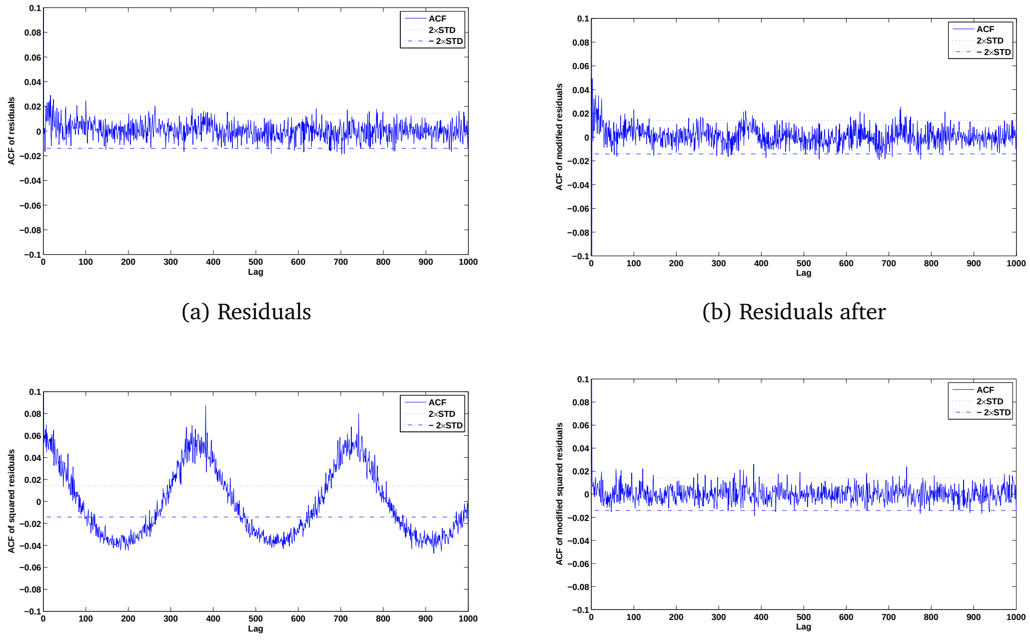


Figure 6: Seasonal variance: daily empirical variance and fitted squared volatility function, represented by the smoothed curve.



(a) Residuals (b) Residuals after

(c) Squared residuals (d) Squared residuals after

Figure 7: ACF Residuals $\widehat{\epsilon_t}$ and squared residuals $\widehat{\epsilon_t}^2$ for the AR(3) during the period 1954.01.01 to 2009.12.31.

where $c_1 = 4.4823$, $c_2 = 2.5635$, $c_3 = 0.7150$, $c_4 = 0.8952$, $c_5 = -0.5473$, $c_6 = 0.3197$, $c_7 = -0.3531$, $c_8 = -0.1315$ and $c_9 = -0.0055$. After dividing $\widehat{\epsilon_t}^2$ by the seasonal variance function σ_t^2, as shown in Fig. 7d we find that the plot of the squared residuals results in much smaller values than before — and moreover, it presents a non-seasonal pattern. Consequently, using the finite difference approximation we obtained values for the coefficients of CAR(3) — viz. $\alpha_1 = 2.0615$, $\alpha_2 = 1.4701$, $\alpha_3 = 0.2955$ (cf. Benth *et al.* [5]).

Since the weather derivative market is not complete, we have to find a risk-neutral probability measure Q equivalent to P. With this risk-neutral probability measure Q and risk-free interest rate r, the arbitrage-free future price of the CAT with the temperature measurement period $[\tau_1, \tau_2]$ is

$$e^{-r(\tau_2-t)}E^Q\left[\int_{\tau_1}^{\tau_2} T_s ds - F_{CAT(t,\tau_1,\tau_2)}\Big|\mathscr{F}_t\right] = 0\,; \tag{3.18}$$

and choosing a probability measure Q_θ characterized by the market price of risk θ, the futures price for the *CAT* with the temperature measurement period $[\tau_1, \tau_2]$ is

$$F_{CAT(t,\tau_1,\tau_2)} = E^{Q_\theta}\left[\int_{\tau_1}^{\tau_2} T_s ds\Big|\mathscr{F}_t\right] , \tag{3.19}$$

Table 7: *CAT* call option prices: Market Price of Risk (MPR)= 0, $r = 0.036$, and the measurement period the whole month of August, with the trading date the first of July.

Exercise time (τ)	K=650	K=700	K=750
25. August 2011	138.64	92.25	45.87
28. August 2011	137.40	91.43	45.46
31. August 2011	136.17	90.61	45.05

where the price of the futures $F_{CAT(t,\tau_1,\tau_2)}$ is $\mathscr{F}_t$-adapted.

In Benth *et al.* [5], explicit formulae for the *CAT* futures price and the call option price are as given in the following Propositions:

Proposition 3.1 (Benth *et al.* [5]). *For $0 \le t \le \tau_1 < \tau_2$, the CAT futures price is*

$$\begin{aligned} F_{CAT(t,\tau_1,\tau_2)} &= \int_{\tau_1}^{\tau_2} \Lambda_u \, du + \mathbf{a}_{t,\tau_1,\tau_2} \mathbf{X}_t + \int_t^{\tau_1} \theta_u \sigma_u \mathbf{a}_{t,\tau_1,\tau_2} \mathbf{e}_p \, du \\ &\quad + \int_{\tau_1}^{\tau_2} \theta_u \sigma_u \mathbf{e}_1^T A^{-1} \Big[\exp[A(\tau_2 - u)] - I_p \Big] \mathbf{e}_p \, du , \end{aligned}$$

where $\mathbf{a}_{t,\tau 1,\tau 2} = \mathbf{e}_1' A^{-1} (\exp(A(\tau_2 - t)) - \exp(A(\tau_1 - t)))$.

Proposition 3.2 (Benth *et al.* [5]). *The price of the CAT call option at $t \le \tau$ is*

$$\begin{aligned} C_{CAT(t,\tau,\tau_1,\tau_2)} &= \exp[-r(\tau - t)] \times \Big\{ \big(F_{CAT(t,\tau_1,\tau_2)} - K\big) \Phi(w(t,\tau,\tau_1,\tau_2)) \\ &\quad + \int_t^{\tau} \Sigma_{CAT}^2(s,\tau_1,\tau_2) \, ds \, \Phi'(w(t,\tau,\tau_1,\tau_2)) \Big\} , \end{aligned}$$

with the strike price K at the exercise time $\tau \le \tau_1$, the measurement period $[\tau_1, \tau_2]$, and w and Σ_{CAT} given by

$$\begin{aligned} w(t,\tau,\tau_1,\tau_2) &= \frac{F_{CAT(t,\tau_1,\tau_2)} - K}{\sqrt{\int_t^{\tau} \Sigma_{CAT}^2(s,\tau_1,\tau_2) \, ds}} , \\ \Sigma_{CAT}(t,\tau_1,\tau_2) &= \sigma(t) \mathbf{e}_1' A^{-1} (e^{A(\tau_2 - t)} - e^{A(\tau_1 - t)}) \mathbf{e}_p . \end{aligned}$$

4. Estimating the Market Price of Risk (MPR)

In Eq. (3.9) and Proposition 3.1, θ represents the market price of risk (MPR). Many researchers [10, 16, 18] have shown that the MPR has a significant effect on pricing options, so it must be determined to calculate the option prices for the *HDD*, *CDD* and *CAT*. In

Table 8: *CAT* call option prices: Market Price of Risk (MPR)=0.0029, $r = 0.036$, and the measurement period the whole month of August, with the trading date the first of July.

Exercise time (τ)	$K = 650$	$K = 700$	$K = 750$
25. August 2011	139.06	92.68	46.29
28. August 2011	137.82	91.85	45.87
31. August 2011	136.58	91.02	45.46

order to estimate the MPR value, information on the actual price for the weather derivatives would be needed if we were to proceed as Härdle & Cabrera [16] did to infer the MPR from the actual option price for Berlin — but in Korea there is no weather market. Consequently, we computed the MPR of the Korea Composite Stock Price Index (KOSPI), and used this value as the MPR for the Korean weather derivatives. Thus the assumed θ is

$$\theta = \frac{\mu - r}{\sigma} ,$$

where r is the risk-free rate, μ is the return, and σ the stock volatility. From the returns on stocks and on 3-year government bonds [14,24], the estimation of the MPR from the KOSPI was -0.0029, and the absolute value of the MPR for temperature could be smaller [18].

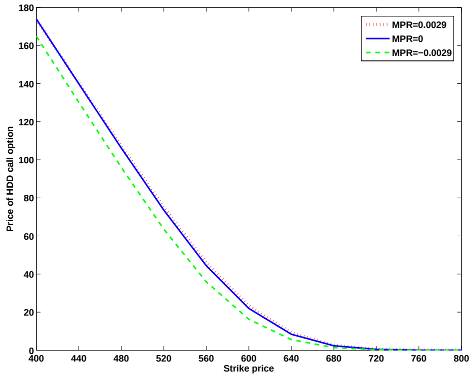
Recalling that β_n and μ_n in Eq. (3.10) depend upon the MPR, we proceeded to obtain the value of θ to use in Eq. (3.9). In our calculations we used the three MPR values 0, -0.0029 and 0.0029, and evaluated the corresponding β_n and μ_n values (denoted by β_n^j and μ_n^j, $j = 0, -, +$) using $\theta = 0$, -0.0029 and 0.0029, respectively. Fig. 8 shows the dependency of the HDD and CDD option prices on the MPR.

Remark 4.1. For the HDD call option, we get the inequalities $\mu_n^- < \mu_n^0 < \mu_n^+$, $\beta_n^+ < \beta_n^0 < \beta_n^-$ and $\mu_n^- - K < \mu_n^0 - K < \mu_n^+ - K$ in Section 3.1; the inequality $HDD_{call}^- < HDD_{call}^0 < HDD_{call}^+$ follows because Φ and the exponential function are monotonic increasing functions — cf. Fig. 8a. The CDD call option is quite similar. Thus since $\mu_n = \sum E^Q[T_{t_i}|\mathscr{F}_t] - 18n$ for the CDD, we obtain $\mu_n^+ < \mu_n^0 < \mu_n^-$, $\beta_n^- < \beta_n^0 < \beta_n^+$ and $\mu_n^+ - K < \mu_n^0 - K < \mu_n^- - K$ such that $CDD_{call}^+ < CDD_{call}^0 < CDD_{call}^-$ — cf. Fig. 8c.

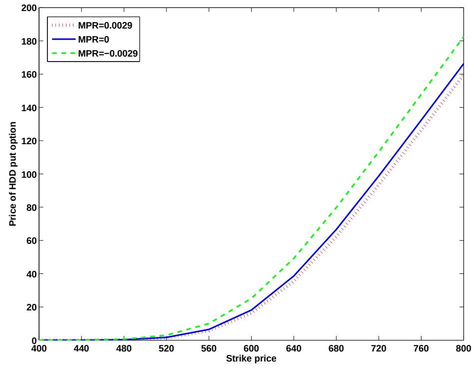
Tables 8 and 9 show the prices of *CAT* call options based on Propositions 3.1 and 3.2 with nonzero MPR. These results imply that the option prices depend on both the exercise time and the MPR, decreasing as the measurement period gets closer or when the MPR is larger.

5. Conclusions

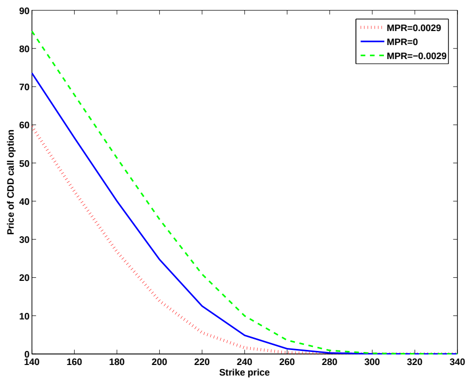
We have extended the temperature model suggested by Alaton *et al.* (2002) and Benth *et al.* (2007) to evaluate option prices for the temperature at Seoul. Using a deterministic model, we price put and call options that are based on the temperature derivatives. To date there are few investigations reported on weather derivatives and their pricing for Asian



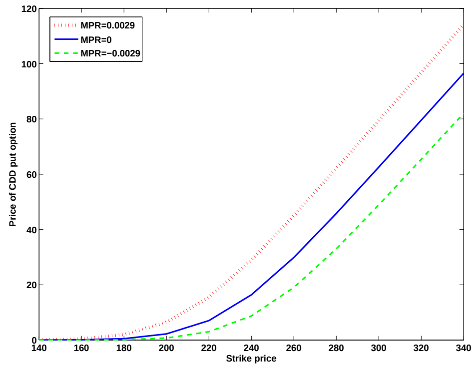
(a) *HDD* call



(b) *HDD* put



(c) *CDD* call



(d) *CDD* put

Figure 8: Option prices. For $r = 0.036$, The calculated *HDD* and *CDD* options for the months of January and August 2011, respectively — the measurement period is the whole month of August, with the trading date being the first of July.

Table 9: *CAT* call option prices. Market Price of Risk (MPR)=-0.0029, $r = 0.036$, and the measurement period is the whole month of August, with the trading date the first of July.

Exercise time (τ)	$K = 650$	$K = 700$	$K = 750$
25. August 2011	138.22	91.83	45.44
28. August 2011	136.98	91.01	45.04
31. August 2011	135.75	90.19	44.63

countries, including Korea. We focused on the temperature for Seoul, but our analysis is readily applicable to other Asian cities.

Since no weather derivatives market exists in Seoul, we considered the market price of risk (MPR) using the Korea Composite Stock Price Index (KOSPI). In particular, we assumed that the MPR is constant, an assumption that deserves more scrutiny in future research — e.g. it may be a piecewise constant or a time-dependent function.

Acknowledgments

This research was partially supported by the research grant NRF-0450-20110002.

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