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New Hybrid Variational Recovery Model for Blurred Images with Multiplicative Noise

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Abstract. A new hybrid variational model for recovering blurred images in the presence of multiplicative noise is proposed. Inspired by previous work on multiplicative noise removal, an I-divergence technique is used to build a strictly convex model under a condition that ensures the uniqueness of the solution and the stability of the algorithm. A split-Bregman algorithm is adopted to solve the constrained minimisation problem in the new hybrid model efficiently. Numerical tests for simultaneous deblurring and denoising of the images subject to multiplicative noise are then reported. Comparison with other methods clearly demonstrates the good performance of our new approach.

AMS subject classifications: 52A41, 65K05, 65K15, 90C25, 90C30
Key words: Convex model, image deblurring, multiplicative noise, Split-Bregman Algorithm, total variation, variational model.

1. Introduction

Image restoration is a classical and important inverse problem in imaging science. In past decades, many restoration methods have been developed for this task — cf. [4, 11, 15, 33, 39, 46] and references therein. In this article, we consider the restoration of blurred images that are also corrupted by multiplicative noise.

Suppose that an image $\hat{u}$ is a real function defined on Ω, a connected bounded open subset of $\mathbb{R}^2$ with compact Lipschitz boundary — i.e. $\hat{u}:\Omega\to\mathbb{R}$. The degraded image f in the presence of simultaneous blurring and multiplicative noise can be represented as

$$f=(A\hat{u})\eta\,, \tag{1.1}$$

where f is positive, $A\in\mathscr{L}(L^2(\Omega))$ is a known bounded linear operator, and η denotes a multiplicative noise with mean one. Multiplicative noise commonly appears in real

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applications such as laser images, ultrasound imaging, synthetic aperture radar (SAR), etc. [5,38,42,44]. Here we specifically focus on multiplicative Gamma noise — i.e. where η follows a Gamma distribution [2,30]. Compared with the denoising case where A is the identity operator, the deblurring poses some extra challenges. This is because image deblurring is an ill-posed problem, due to either the possible nonuniqueness of the solution or numerical instability induced by the operator A [15]. In order to overcome these problems, several variational models with regularisation have been proposed, based on the image degradation model and prior information on $\hat{u}$.

Indeed, according to the statistical properties of η, the recovery of the image $\hat{u}$ may be achieved by solving the constrained minimisation problem [38]

$$\begin{aligned} &\inf_{u\in S(\Omega)} \int_\Omega |Du| \\ &\text{subject to} \int_\Omega f/(Au)\,dx = 1 \text{ and } \int_\Omega [f/(Au)-1]^2\,dx = \theta^2\,, \end{aligned} \tag{1.2}$$

where θ is the standard deviation of η and $S(\Omega) = \{v \in BV(\Omega) : v > 0\}$. Here $BV(\Omega)$ denotes the space of functions of bounded variation (i.e. $u \in BV(\Omega)$ if and only if $u \in L^1(\Omega)$), and the BV-seminorm

$$\int_\Omega |Du| := \sup\left\{\int_\Omega u \cdot \operatorname{div}(\xi(x))dx : \xi \in C_0^\infty(\Omega,\mathbb{R}^2), \|\xi\|_{L^\infty(\Omega,\mathbb{R}^2)} \le 1\right\} \tag{1.3}$$

is finite. The space $BV(\Omega)$ endowed with the norm $\|u\|_{BV} = \|u\|_{L^1} + \int_\Omega |Du|$ is a Banach space. If $u \in BV(\Omega)$, the distributional derivative Du is a bounded Radon measure and the term $\int_\Omega |Du|$ defined in (1.3) corresponds to the total variation (TV). Based on the compactness of $BV(\Omega)$, in the two-dimensional case one has the embedding $BV(\Omega) \hookrightarrow L^p(\Omega)$ for $1 \le p \le 2$, compact for $p < 2$ — cf. [1,3,15] for more detail.

In the model (1.2), which we call the RLO model, the TV of u is applied as the objective function in order to preserve edge information in the images. Only basic statistical properties of the noise η (viz. the mean and the variance) are involved in (1.2), which slightly limits the quality of the restored images. Consequently, based on *maximum a posteriori* (MAP) analysis of the multiplicative Gamma noise, Aubert & Aujol [2] proposed the following variational model for image deblurring under multiplicative noise, which we refer to as the AA model:

$$\inf_{u\in S(\Omega)} \int_\Omega \left(\log(Au) + \frac{f}{Au}\right) dx + \lambda \int_\Omega |Du|\,, \tag{1.4}$$

where the TV of u is again used as the regularisation term and $\lambda > 0$ is the regularisation parameter that controls the trade-off between a good fit of f and smoothness due to the TV regularisaton. Since both the RLO model (1.2) and the AA model (1.4) are non-convex, the gradient projection algorithms proposed in Refs. [2,38] may lead to certain local minimisers, so the quality of the corresponding restoration results is strongly dependent on the initial estimations of $\hat{u}$ and the numerical optimisation procedures used.