East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.141214.050616a

Vol. **6**, No. 4, pp. 384-399
November 2016

Dynamics of a Modified Predator-Prey System to allow for a Functional Response and Time Delay

Wei Liu[1,2] and Yaolin Jiang[1,*]

[1] *School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China.*
[2] *School of Mathematics and Computer Science, Xinyu University, Xinyu 338004, China.*

Received 14 December 2014; Accepted (in revised version) 5 June 2016.

Abstract. A modified predator-prey system described by two differential equations and an algebraic equation is discussed. Formulae for determining the direction of a Hopf bifurcation and the stability of the bifurcating periodic solutions are derived differential-algebraic system theory, bifurcation theory and centre manifold theory. Numerical simulations illustrate the results, which includes quite complex dynamical behaviour.

AMS subject classifications: 92D25

Key words: Predator-prey system, harvesting, stability, time delay, periodic solutions.

1. Introduction

There has been wide-ranging interest in the modelling of predator-prey systems with harvesting [1, 2, 4, 5, 9–22]. Some type of time delay has often been incorporated into predator-prey systems to account for maturation time, capturing time, or some other factor. Indeed, it is well recognised that time delay can play an important role in many biological dynamical systems. Associated delay differential equations usually exhibit much more complicated dynamics than ordinary differential equations, since a time delay can cause a stable equilibrium to become unstable or cause the population to fluctuate [10, 13, 19]. Refs. [4, 10, 14] proposed the following predator-prey system with time delay:

$$\begin{cases} \dot{x}(t) = x(t)G(x(t-\tau)) - P(x(t))y(t)\,, \\ \dot{y}(t) = y(t)\big[-a + dP(x(t))\big], \end{cases}$$

where $x(t)$ and $y(t)$ denote the respective population densities of the prey and predator at time t, $a > 0$ is the death rate of predator, $d > 0$ is the conversion factor, the delay $\tau \geq 0$

*Corresponding author. *Email addresses:* wliu2015@163.com (W. Liu), yljiang@mail.xjtu.edu.cn (Y. Jiang)

can be regarded as a gestation period or reaction time of the prey, $G(x(t))$ is the intrinsic growth rate, and $P(x(t))$ is the functional response. Here we choose a traditional logistic form $G(x(t)) = r - kx(t)$, where $r > 0$ is the growth rate of the prey in the absence of the predator and $k > 0$ represents the self-regulation constant of the prey, and $P(x(t)) = \sqrt{x(t)}$ corresponding to the prey interacting with the predator along the outer margin of the herd [5, 9, 15, 16]. Thus we consider the following predator-prey system with time delay:

$$\begin{cases} \dot{x}(t) = x(t)\left(r - kx(t-\tau) - \dfrac{y(t)}{\sqrt{x(t)}} - E(t)\right), \\ \dot{y}(t) = y(t)\left(-a + d\sqrt{x(t)}\right). \end{cases} \tag{1.1}$$

Gordon [6] studied the effect of harvesting on an ecosystem from an economic perspective, and proposed the economic principle

$$\text{Net Economic Revenue (NER)} = \text{Total Revenue (TR)} - \text{Total Cost (TC)}\,.$$

For the system (1.1), an algebraic equation that represents the economic profit v of the harvesting effort on the prey is

$$E(t)(px(t) - c) = v\,, \tag{1.2}$$

where $E(t)$ denotes the harvesting effort for the prey, and $p > 0$, $c > 0$ and $v > 0$ are respectively the harvesting reward per unit harvesting effort for a unit weight of prey, the harvesting cost per unit harvesting effort for the prey and the economic profit per unit harvesting effort. Consequently, on combining (1.1) and (1.2) we obtain the following modified predator-prey system with functional response $\sqrt{x}$ and time delay:

$$\begin{cases} \dot{x}(t) = x(t)\left(r - kx(t-\tau) - \dfrac{y(t)}{\sqrt{x(t)}} - E(t)\right), \\ \dot{y}(t) = y(t)\left(-a + d\sqrt{x(t)}\right), \\ 0 = E(t)(px(t) - c) - v\,. \end{cases} \tag{1.3}$$

The effect of varying the time delay on the dynamics of the system (1.3) in the region $\mathrm{R}^3_+ = \{(x, y, E)|\ x > 0,\ y > 0,\ E > 0\}$ is the main focus of this article. In research on the dynamic behaviour of similar predator-prey systems, issues such as saddle-node bifurcation, singularity induced bifurcation, Neimark-Sacker bifurcation and state feedback control have been discussed [12, 20–22]. However, formulae for determining the properties of possible Hopf bifurcation have not yet appeared. By applying a local parameterisation method [3] and centre manifold theory [8], here we investigate the stability and the direction of the Hopf bifurcation of the delayed system (1.3) where τ is a bifurcation parameter, to be defined below. In passing, it is notable that t is occasionally omitted whenever there is no danger of confusion.