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A Relaxation Two-Sweep Modulus-Based Matrix Splitting Iteration Method for Linear Complementarity Problems

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Abstract. A general RTMS iteration method for linear complementarity problems is proposed. Choosing various pairs of relaxation parameters, we obtain new two-sweep modulus-based matrix splitting iteration methods and already known iteration procedures such as the MS [1] and TMS [27] iteration methods. If the system matrix is positive definite or an H_+-matrix and the relaxation parameters ω_1 and ω_2 satisfy the inequality $0 \le \omega_1, \omega_2 \le 1$, sufficient conditions for the uniform convergence of MS, TMS and NTMS iteration methods are established. Numerical results show that with quasi-optimal parameters, RTMS iteration method outperforms MS and TMS iteration methods in terms of computing efficiency.

AMS subject classifications: 65F10, 65F35, 65H10

Key words: Linear complementarity problem, matrix splitting, iteration method, relaxation, convergence.

1. Introduction

Let $\mathbf{R}^n$ and $\mathbf{R}^{n\times n}$ be, respectively, the n-dimensional real vector and matrix spaces. For a matrix $A \in \mathbf{R}^{n\times n}$ and a vector $q \in \mathbf{R}^n$, the linear complementarity problem, abbreviated as LCP(q,A), consists in finding the pair of vectors $w, z \in \mathbb{R}^n$, such that

$$w := Az + q \ge 0, \quad z \ge 0 \quad \text{and} \quad z^{\mathsf{T}} w = 0, \tag{1.1}$$

where T denotes the transposition operation. The LCP(q,A) often arises in applications such as free boundary problems, network equilibrium, contact problems — cf. [7, 10, 22] and the references therein. To solve the LCP(q,A), van Bokhoven [24] reformulated it as an implicit fixed-point equation. The procedure, called the modulus method, was modified by Dong and Jiang [9] by including a shifting parameter into iteration process. Bai [1]

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established a modulus-based matrix splitting (MS) iteration method, based on a more effective and economical matrix splitting technique in actual computation. Zhang and Ren [29] proved the convergence of MS iteration method under a weak condition and Li [15] considered MS iteration method under more general conditions. The accelerated overrelaxation types of MS iteration methods have been studied in [8, 12] and the best diagonal matrix-parameter for such approaches has been determined. Further generalisations of the modulus-based matrix splitting iteration methods are connected with special matrix splittings [14, 20, 30, 32], preconditioning technique [16, 28] or relaxation strategies [26,31]. Moreover, the modulus-based synchronous multisplitting and modulus-based synchronous two-stage multisplitting iteration methods, aimed at the high parallel computational efficiency are developed and analysed [2, 3, 17]. On the other hand, modulus-based MS iteration methods have been applied to nonlinear complementarity problems — cf. [13, 18, 19, 21].

Here, starting from the two-sweep modulus-based matrix splitting (TMS) iteration method in Ref. [27], we consider a relaxation two-sweep modulus-based matrix splitting iteration method and prove its convergence for H_+ and positive definite system matrices, where relaxation strategy is different from [31]. This new method includes MS iteration method [1] and TMS iteration method [27] as its special cases and contains new two-sweep modulus-based matrix splitting iteration methods. Moreover, numerical results show its superiority over similar methods, both in number of iterations and CPU time.

The rest of this paper is organised as follows. In Section 2 we provide necessary definitions and auxiliary results. The relaxation two-sweep modulus-based matrix splitting iteration method is introduced in Section 3 and its convergence is studied in Section 4. Numerical results are discussed in Section 5. Section 6 contains concluding remarks.

2. Preliminaries

Most of material presented in this sections can be found in Refs. [6, 7, 11, 22, 25].

If $A = (a_{ij})$ and $B = (b_{ij})$ are real $m \times n$ matrices, then the inequality $A \geq B$ $(A > B)$ means that $a_{ij} \geq b_{ij}$ $(a_{ij} > b_{ij})$ for all i and j. Subsequently, matrix $A = (a_{ij})$ is called non-negative (positive) if $a_{ij} \geq 0$ $(a_{ij} > 0)$ for all i and j. Besides, for any $A \in \mathbb{R}^{m\times n}$ by $|A|$ we denote the matrix $(|a_{ij}|)$.

Let A be a square matrix and $\mathrm{sp}\,(A)$ refer to the spectrum, $\rho(A)$ to the spectral radius and $\mathrm{diag}(A)$ to the diagonal part of A. Moreover, the comparison matrix $\langle A\rangle = (\langle a_{ij}\rangle)$ for A is the one with the entries $\langle a_{ij}\rangle = |a_{ij}|$ if $i = j$ and $\langle a_{ij}\rangle = -|a_{ij}|$ if $i \neq j$. The matrix A is called Z-matrix if all off-diagonal entries of A are non-positive, M-matrix if it is a Z-matrix with $A^{-1} \geq 0$ and H-matrix if its comparison matrix $\langle A\rangle$ is an M-matrix. Besides, any H-matrix with positive diagonal entries is called H_+-matrix.

The representation $A = M - N$ is called:

1. The splitting of the matrix A if M is a nonsingular matrix.

2. Convergent splitting if $\rho(M^{-1}N) < 1$.