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Maxwell Exterior Transmission Eigenvalue Problems and Their Applications to Electromagnetic Cloaking

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Abstract. Maxwell exterior transmission eigenvalue problems arising in scattering problem for a penetrable cavity are considered. Properties of such eigenvalues for a weak formulation of the transmission problem in unbounded domains are studied and the Calderon operator for Maxwell's system is described. In particular, the absence of pure imaginary eigenvalues and the discreteness of the set of exterior transmission eigenvalues are established. Applications to exterior invisibility cloaking are also considered. Using the Maxwell-Herglotz approximation, we can generate nearly non-scattering waves corresponding to the exterior eigenvalues of an electromagnetic medium.

AMS subject classifications: 35P25, 35Q60

Key words: Exterior transmission eigenvalue problem, Calderon operator, exterior invisibility cloaking.

1. Introduction

The transmission eigenvalue problem plays an important role in inverse scattering theory [1, 2, 6, 7, 9, 12, 16–19, 23, 25, 35]. As the result, it attracted a wide attention from numerical analysts [3,5,8,10,13,15,24]. Transmission eigenvalues are connected with the inverse medium problem, which consists in the reconstruction of refractive index from far or near field patterns of a scattered wave. In particular, the scattering of incident plane waves by a bounded non-absorbing medium is closely connected to the interior transmission eigenvalue problem and has been vigorously studied — cf. [8,33,34]. This problem is formulated in the form of two elliptic equations in a bounded domain with the same Cauchy

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data on the boundary. On the other hand, the exterior transmission problem arising in the inverse scattering of cavities surrounded by non-absorbing medium [30–32], is not well studied. The motivation for investigating such kind of problems is guided by the problem of oil transportation — viz. we want to determine the internal wall of an underground pipeline by using receivers and transmitters in oil. The interior transmission eigenvalue problem has been recently used in the development of invisible cloaking devices for the scattering measurements of waves generated by the set of interior transmission eigenvalues [13, 21]. Here, we exploit the exterior transmission eigenvalue problem to study the exterior invisibility. Note that it is a non-self adjoint problem in an unbounded domain.

We consider time-harmonic electromagnetic wave scattering for a cavity surrounded by a penetrable inhomogeneous medium of compact support. Let $\mathbb{M}^{3\times 3}_{\mathrm{sym}}$ denote the space of real-valued symmetric matrices and $\Omega \subset \mathbb{R}^3$ be a Lipschitz domain. Any function $\gamma \in L^\infty(\Omega; \mathbb{M}^{3\times 3}_{\mathrm{sym}})$ such that

$$c_0|\xi|^2 \le \gamma(x)\xi \cdot \xi \le c_0^{-1}|\xi|^2 \quad \text{for} \quad a.e. \quad x \in \Omega \quad \text{and every} \quad \xi \in \mathbb{R}^3,$$

where $0 < c_0 < 1$ is a constant, is called the tensor in Ω satisfying the uniform ellipticity condition. In what follows, c_0 is referred to as the ellipticity constant of the tensor γ.

Let $(\Omega; \epsilon, \mu, \sigma)$ be an electromagnetic medium (EM) occupying a domain $\Omega \subset \mathbb{R}^3$ and having electric permittivity ϵ, magnetic permeability μ and electric conductivity σ. We say that $(\Omega; \epsilon, \mu, \sigma)$ is regular if $\epsilon, \mu, \sigma \in L^\infty(\Omega; \mathbb{M}^{3\times 3}_{\mathrm{sym}})$, the tensors ϵ and μ are uniformly elliptic and there is a constant $\lambda_0 \in \mathbb{R}^+$ such that

$$0 \le \sigma(x)\xi \cdot \xi \le \lambda_0|\xi|^2 \quad \text{for} \quad a.e. \quad x \in \mathbb{R}^3 \quad \text{and every} \quad \xi \in \mathbb{R}^3.$$

A function $\gamma(x)$, $x \in \Omega$ is called isotropic if there exists an $\alpha(x) \in L^\infty(\Omega; \mathbb{R})$ such that $\gamma(x) = \alpha(x) \cdot \mathbf{I}_{3\times 3}$, where $\mathbf{I}_{3\times 3}$ is the 3×3 identity matrix.

Suppose that $(D; \epsilon, \mu, \sigma)$ is an isotropic homogeneous cavity with the parameters ϵ, μ scaled to be 3×3 identity matrices and σ to be zero in D, which are the same as the material parameters for the medium outside of a large enough ball $B_R \supset D$. The medium in $B_R \backslash D$ is inhomogeneous — i.e.

$$\epsilon(x) \not\equiv \mathbf{I}_{3\times 3}, \quad \mu(x) \not\equiv \mathbf{I}_{3\times 3}, \quad \sigma(x) \not\equiv 0, \quad x \in B_R \backslash D.$$

Fix an EM wavenumber $k \in \mathbb{R}^+$ in EM spectrum and consider the EM radiation in this frequency regime in the space

$$\left(\mathbb{R}^3; \epsilon, \mu, \sigma\right) = \left(D; \mathbf{I}_{3\times 3}, \mathbf{I}_{3\times 3}, 0\right) \cup \left(\mathbb{R}^3 \backslash \overline{D}; \epsilon, \mu, 0\right). \tag{1.1}$$

Let $\mathbf{E}^i$ be the incident electric field represented in the form of the electric dipole — i.e.

$$\mathbf{E}^i = \mathbb{G}(x, z)\mathbf{p} := \frac{i}{k} \mathrm{curl}_x \mathrm{curl}_x \Phi(x, z)\mathbf{p},$$

where $\mathbb{G}(x, z)$ is Green's tensor, $\mathbf{p}$ the polarisation and

$$\Phi(x, z) = \frac{1}{4\pi} \frac{e^{ik|x-z|}}{|x-z|}$$