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STRONG CONVERGENCE AND STABILITY OF THE SEMI-TAMED AND TAMED EULER SCHEMES FOR STOCHASTIC DIFFERENTIAL EQUATIONS WITH JUMPS UNDER NON-GLOBAL LIPSCHITZ CONDITION

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Abstract. We consider the explicit numerical approximations of stochastic differential equations (SDEs) driven by Brownian process and Poisson jump. It is well known that under non-global Lipschitz condition, Euler Explicit method fails to converge strongly to the exact solution of such SDEs without jumps, while implicit Euler method converges but requires much computational efforts. We investigate the strong convergence, the linear and nonlinear exponential stabilities of tamed Euler and semi-tamed methods for stochastic differential equation driven by Brownian process and Poisson jumps, both in compensated and non compensated forms. We prove that under non-global Lipschitz condition and superlinearly growing drift term, these schemes converge strongly with the standard one-half order. Numerical simulations to substain the theoretical results are provided.

Key words. Stochastic differential equation, strong convergence, linear stability, exponential stability, jump processes, one-sided Lipschitz.

1. Introduction

In this work, we consider jump-diffusion Itô's stochastic differential equations (SDEs) of the form in the interval $[0, T]$

$$dX(t) = f(X(t^-))dt + g(X(t^-))dW(t) + h(X(t^-))dN(t), \quad X(0) = X_0. \tag{1}$$

Here $W(t)$ is a m-dimensional Brownian motion, $f : \mathbb{R}^d \longrightarrow \mathbb{R}^d$, $d \in \mathbb{N}$ satisfies the one-sided Lipschitz condition and the polynomial growth condition, the functions $g : \mathbb{R}^d \longrightarrow \mathbb{R}^{d\times m}$ and $h : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ satisfy the globally Lipschitz, and $N(t)$ is a one dimensional Poisson process with parameter λ. Extension to vector-valued jumps with independent entries is straightforward. The one-sided Lipschitz function f can be decomposed as $f = u + v$, where the function $u : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ is the global Lipschitz continuous part and $v : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ is the non-global Lipschitz continuous part, see e.g. [25]. Using this decomposition, we can rewrite the jump-diffusion SDEs (1) in the following equivalent form

$$X(t) = \left(u(X(t^-) + v(X(t^-))\right) dt + g(X(t^-))dW(t) + h(X(t^-))dN(t). \tag{2}$$

This decomposition will be used only for semi-tamed schemes. Equations of type (1) arise in a range of scientific, engineering and financial applications [3, 1, 14]. Most of such equations do not have explicit solutions and therefore one requires numerical schemes for their approximations. Their numerical analysis has been studied in [6, 24, 5, 19] with implicit and explicit schemes where strong and weak convergence have been investigated. The implementation of implicit schemes requires significantly more computational effort than the explicit Euler-type approximations as Newton method is usually required to solve nonlinear systems at each time iteration in implicit schemes. The standard explicit method for approximating SDEs of type (1) is the Euler-Maruyama method [19]. Recently it has been proved

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(see [13, 11]) that the Euler-Maruyama method often fails to converge strongly to the exact solution of nonlinear SDEs of the form (1) without jump term when at least one of the functions f and g grows superlinearly. To overcome this drawback of the Euler-Maruyama method, numerical approximation, with computational cost close to that of the Euler-Maruyama method and which converges strongly even in the case the function f is superlinearly growing was first introduced in [12] and strong convergence was investigated. Further investigations have been performed in the litterature (see for example [21, 9, 25] and references therein), where in [21] the time step Δt in [12] is replaced by its power Δt^{α}, $\alpha \in (0, 1/2]$ in the denominator of the taming drift term. Recently the work in [21] has been extended for SDEs driven by compensated Levy noise in [2, 15]. The condition $\alpha \in (0, 1/2]$ is key in the convergence proofs in [2, 21, 15], so the proofs cannot be extended for $\alpha \in [1/2, 1]$. Strong and weak convergences are not the only features of numerical techniques. Stability is also a good feature as the information about time step size for which does a particular numerical method replicate the stability properties of the exact solution is valuable. The linear stability is an extension of the deterministic A-stability while exponential stability can guarantee that errors introduced in one time step will decay exponentially in future time steps, exponential stability also implies asymptotic stability [8]. By the Chebyshev inequality and the Borel–Cantelli lemma, it is well known that exponential mean-square stability implies almost sure stability [8]. The stability of classical implicit and explicit methods for (1) are well understood [6, 8, 24]. Although the strong convergence of tamed schemes with and without jump have been studied, a rigorous stability properties have not yet been investigated to the best of our knowledge.

The aim of this paper is to study the strong convergence of tamed schemes driven by Brownian process and Poisson jump for $\alpha \in [1/2, 1]$, and to provide a rigorous study of the linear and exponential stabilities of semi-tamed and tamed schemes for $\alpha \in [0, 1]$. Following closely the breakthrough idea in [12], we provide the strong convergence of the tamed schemes and the corresponding semi tamed schemes both in compensated and non compensated forms for $\alpha \in [1/2, 1]$. The extensions are not straightforward as several technical lemmas are needed. Numerical experiments show that the semi-tamed works better than the tamed and compensated tamed schemes. Numerical results also show that the tamed and the compensated tamed Euler scheme have good stability behavior when α approaches 1. Therefore, our tamed schemes with $\alpha \in [1/2, 1]$ have better stability property than the tamed schemes presented in [2] for $\alpha \in (0, 1/2]$.

The paper is organized as follows. Section 2 presents the classical result of existence and uniqueness of the solution X of (1). The compensated and non compensated tamed schemes and semi-tamed scheme are presented in Section 3 along with their strong convergences. The linear stability of the schemes is provided in Section 4 while the nonlinear exponential stability is provided in Section 5. We end in Section 6 by providing some numerical simulations.

2. Notations, assumptions and well posedness

Throughout this work, $(\Omega, \mathcal{F}, \mathbb{P})$ denotes a complete probability space with a filtration $(\mathcal{F}_t)_{t\geq 0}$. For all $x, y \in \mathbb{R}^d$, we denote by $\langle x, y\rangle = x_1y_1 + x_2y_2 + \cdots + x_dy_d$, $\|x\| = \langle x, x\rangle^{1/2}$, $\|A\| = \sup\limits_{x\in\mathbb{R}^d, \|x\|\leq 1} \|Ax\|$ for all $A \in \mathbb{R}^{m\times d}$. $a\vee b$ represents $\max\{a, b\}$. We use also the following convention : $\sum_{i=u}^{n} = 0$ for $u > n$.

We first ensure that SDEs (1) is well-posed. The following assumption is needed.