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APPLICATION OF PARALLEL AGGREGATION-BASED MULTIGRID TO HIGH RESOLUTION SUBSURFACE FLOW SIMULATIONS

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Abstract. In this paper we assess the parallel efficiency issues for simulating single phase subsurface flow in porous media, where the permeability tensor contains anisotropy rotated with certain angles or severe discontinuity. Space variables are discretized using multi-points flux approximations and the pressure equations are solved by aggregation-based algebraic multigrid method. The involved issues include the domain decomposition of space discretization and coarsening, smoothing, the coarsest grid solving of multigrid solving steps. Numerical experiments exhibit that the convergence of the multigrid algorithm suffers from the parallel implementation. The linear system at the coarsest grid is solved and by various iterative methods and the experimental results show that the parallel efficiency is less attenuated when sparse approximate inverse preconditioning conjugate gradient is used.

Key words. Parallel computation, porous media flow, multi-points flux approximations, algebraic mulitgrid.

1. Introduction

Modeling of subsurface flow processes is important for many applications, for example, groundwater resource management, CO_2 sequestration, and petroleum production. In all the cases, fluids flow through medium containing pores (voids). A porous media is most often characterized by porosity and the media's ability to transmit fluids is measured by a quantity called permeability. To describe the physics of the fluid flow, the Darcy's law is used to establish the partial differential equations, where the geometrical complexities of porous media is replaced by one parameter relates to pressure gradient vector and the fluid flux vector. As a consequence of the different geologic processes over geologic time-scales, however, there are cases such that the rock formations is geometric complicate in a way the hydraulic properties of media are mostly heterogeneous and anisotropic, where the flux vector does not coincide the hydraulic gradient vector.

The appearance of anisotropy in permeability which is not aligned with the direction of principal axes imposes challenges on the development of reservoir simulator and the stencil arising from the standard two-point flux approximation (TPFA) fails to account for the fact that pressure gradient in one direction can cause flow in other directions as well. This motivates the development of multi-point flux approximation (MPFA) [1, 2], which is designed based on the finite volume formulation. MPFA introduces the surface midpoints through the interaction region to ensure the pressure and flux continuities and the flux is calculated using informations involving several points as opposed to only two points. This allows the application of MPFA methods on general nonorthogonal grids as well as for general orientation of

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the principal directions of the permeability tensor. In the work of Aavatsmark et al. [1], MPFA is introduced with two subclasses, i.e., MPFA O- and U- methods, based on the choice of continuity conditions. In [3] Aavatsmark discussed the discretization and implementation of MPFA-O method with subsurface midpoint as continuity points on quadrilateral grids. In addition, Edwards and Rogers [2] have demonstrated the existence of an error produced by the classical five-point stencil scheme (TPFA) due to the ignorance of the off-diagonal elements in the tensor. The off-diagonal elements of the tensor equation could not be ignored since these give a strong impact on the variation of the pressure field. Besides the mentioned subclasses, there are other types of MPFA such as MPFA L-method [4] and MPFA Z-method [5].

The linear systems arising from MPFA methods can be solved efficiently by direct methods (Gauss eliminations) when the number of space cells is small [6, 7]. In recent years, however, the study of flow phenomena in multiple scales (e.g., from pore scale to Darcy scale) and in fractured porous media have drawn significant attention [8, 9, 10, 11]. In these research the mesh with high resolution is needed in order to resolve wide range of length scale of interest and solving the associated linear system using direct method is not favourable. As the resulting matrix from MPFA being sparse, iterative methods turn out to be better options. Among the iterative methods, multigrid methods are well known for being fastest numerical methods for solving linear systems arising from discretization of elliptic partial differential equations (PDEs) [12]. For large classes of problems it can be shown that the convergence rate of multigrid method will not deteriorate as the mesh size increases and the total work to solve a linear system with N variables is $O(N)$ if a fixed level of accuracy is needed [13, 14].

In this paper, we incorporate an aggregation-based algebraic multigrid (AGMG) method [15] in solving subsurface flow equations. The cases studied include the TPFA discretized pressure equations in single-phase flow where the permeability contains discontinuity with large jump and rotated anisotropy. On the other hand, we consider the linear system arising from MPFA discretization. Furthermore, the parallelization of the solution algorithm is implemented and carried out in massively parallel simulations. Various issues and difficulties are addressed and discussed. The remaining part of the paper is organized as follows. In Section 2 we state the mathematical model of fluid flow in porous media, MPFA method and experimenting field approach. In Section 3 we discuss several issues for parallelizing multigrid methods. In Section 4 the setup of numerical experiments and related facility are discussed. Results of several numerical experiments are presented in Section 5.

2. Subsurface flow in anisotropic porous media

2.1. Governing equations. The governing equations of fluid flow in porous media are given by the combination of physical principles: the conservation of mass and Darcy's law. The principle of mass conservation assumes the mass inflow and outflow are equal when fluid flow crosses a certain region. Thus the mass conservation equation is given by

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \mathbf{q}, \tag{1}$$