INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 17, Number 3, Pages 316–331

CS-MRI RECONSTRUCTION BASED ON THE CONSTRAINED TGV-SHEARLET SCHEME

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Abstract. This paper proposes a new constrained total generalized variation (TGV)-shearlet model to the compressive sensing magnetic resonance imaging (MRI) reconstruction via the simple parameter estimation scheme. Due to the non-smooth term included in the proposed model, we employ the alternating direction method of multipliers (ADMM) scheme to split the original problem into some easily solvable subproblems in order to use the convenient soft thresholding operator and the fast Fourier transformation (FFT). Since the proposed numerical algorithm belongs to the framework of the classic ADMM, the convergence can be kept. Experimental results demonstrate that the proposed method outperforms the state-of-the-art unconstrained reconstruction methods in removing artifacts and achieves lower reconstruction errors on the tested dataset.

Key words. Magnetic resonance imaging, total generalized variation, shearlet transformation, alternating direction method of multipliers (ADMM), compressive sensing.

1. Introduction

Magnetic resonance imaging (MRI) is commonly used in radiology to visualize the internal structure and function of the body by noninvasive and nonionizing means. However, the widespread use of MRI is hindered by its intrinsic slow data acquisition process. So how to speed up the scanning time has been the key in the MRI research community. Recently, compressive sensing (CS) [3] has shown great potential in speeding up MRI by under-sampling k-space data. In the meantime, reducing the acquired data which compromises with its diagnostic value may result in degrading the image quality. Considering the above reasons, finding an inversion algorithm with good practical performance in terms of image quality and reconstruction speed is crucial in clinical applications.

Let u be an ideal image scaled in $[0, 1]$ and set $A = P\mathcal{F}$, where P is a selection matrix and $\mathcal{F}$ is the Fourier transformation matrix. Accordingly, the undersampling k-space data f involved in the sampling matrix A and the additive noise η can be boiled down to

$$f = Au + \eta. \tag{1}$$

From the view of the numerical computation, reconstructing u from f is an ill-posed problem since the operator A depends on imaging devices or data acquisition patterns, which usually leads to a large and ill-conditioned matrix. So some variational-PDE based models have been proposed to overcome these drawbacks.

In order to improve the scanning time of the variational-PDE based models, motivated by the compressed sensing (CS) theory, Lustig et al. [29] proposed an unconstrained model to reconstruct CS-MRI images as follows:

$$\min_u \|u\|_{TV} + \tau \left\|\Phi^\top u\right\|_1 + \frac{\eta}{2}\|Au - f\|_2^2, \tag{2}$$

Received by the editors July 8, 2018 and, in revised form, March 11, 2020.
2000 *Mathematics Subject Classification.* 90C25, 49M27, 68U10, 94A08.

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where $\|u\|_{TV} = \|\nabla u\|_1$ is the total variation [5, 7, 24, 35, 43, 48] (TV), Φ is the wavelet transformation, the superscript $\top$ denotes (conjugate) transpose of matrix. $\|\Phi^\top u\|_1$ is the ℓ_1-norm of the representation of u under the wavelet transformation Φ, $\tau > 0$ is a scalar which balances Φ sparsity and TV sparsity.

As we know, the TV-based regularization in the model (2) can handle few-views problems in the MRI reconstruction, which has the advantage to preserve edges when removing noises in homogeneous regions. However, it usually tends to cause staircase-like artifacts [22, 26, 28, 32, 35] due to their nature of favoring piecewise constant solutions. To alleviate the above drawbacks, the total generalized variation (TGV) in [2] has attracted much interest in image science. On the other hand, the continuous wavelet transformation of a distribution f decays rapidly near the points where f is smooth, while it decays slowly near the irregular points. This property allows the identification of the singular support of f. However, the continuous wavelet transformation is unable to describe the geometry of the singularity set of f and, in particular, to identify the wavefront set of a distribution [40]. Unlike the traditional wavelets used in the second regularized term of (2) lacking the ability to detect directionality, the shearlets provide a multidirectional as well as a multiscale decomposition for multi-dimension signals [17, 18]. There are two main advantages of using shearlets regularization in reconstruction: one is that shearlets allow for a lower redundant sparse tight frame representation than other related multiresolution representations, while still offering shift invariance and a directional analysis; another is that the shearlet representation can be used to decompose the space $L^2(\Omega)$ of images into a sequence of spaces, while we apply the soft thresholding operator conveniently to numerical algorithm. Obviously, shearlets are better candidates than wavelets, as shearlets have essentially optimal approximation errors for images that contain edges apart from discontinuities along curves. So following these observations, Guo et al. [19] coupled the TGV with the shearlet transformation to reconstruct high quality images from incomplete compressive sensing measurements as

$$\min_u \mathrm{TGV}_\alpha^2(u) + \beta \sum_{j=1}^{N} \|\mathcal{SH}_j(u)\|_1 + \frac{\nu}{2}\|Au - f\|_2^2, \tag{3}$$

where $\mathcal{SH}_j(u)$ is the jth subband of the shearlet transformation of u; $\beta > 0$ balances the shearlet transformation sparsity and the TGV sparsity; $\nu > 0$ is the regularization parameter.

In the model (3), the key is how to balance two parameters β and ν. In form, an improperly large weight for the data fidelity term results in serious residual artifacts, whereas an improperly small weight results in damaged edges and fine structures [8]. To overcome these drawbacks, it needs to turn to the following constrained optimization model as

$$\begin{array}{ll} \min\limits_u & \mathrm{TGV}_\alpha^2(u) + \beta \sum\limits_{j=1}^{N} \|\mathcal{SH}_j(u)\|_1 \\ \text{s.t.} & \|Au - f\|_2 \le \sigma, \end{array} \tag{4}$$

where σ implies some prior information of noise. Compared with unconstrained model (3), the model (4) can estimate the noise level σ more easily than finding a suitable parameter ν. These two models are equivalent in nature when choosing suitable penalty parameter ν. In fact, this equivalency transformation has been successfully applied to imaging and sparsity tasks [37, 41, 42, 45] for the