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EVEN-ODD CYCLED HIGH-ORDER SPLITTING FINITE DIFFERENCE TIME DOMAIN METHOD FOR MAXWELL'S EQUATIONS

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Abstract. In the paper, an even-odd cycled high-order splitting finite difference time domain scheme for Maxwell's equations in two dimensions is developed. The scheme uses fourth order spatial difference operators and even-odd time step technique to make it more accurate in both space and time. The scheme is energy-conserved, unconditionally stable and efficient in computation. We analyze in detail the stability, dispersion and phase error for the scheme. We prove that the scheme is energy conservative. Numerical experiments show numerically the energy conservation, high accuracy, and the divergence free accuracy. Furthermore, the developed scheme is applied to compute of the grounded coplanar waveguides.

Key words. Maxwell's Equations, even-odd cycled, high order in time, dispersion analysis, energy conservation, grounded coplanar waveguide.

1. Introduction

Maxwell's equations are widely used in computational electromagnetism applications such as, radio frequency, microwave, antennas, and air-craft radars and so on. Several ADI and splitting finite difference time domain methods have been developed to compute the solutions of Maxwell's equations. Second order schemes are commonly used for moderate numerical results, however, high order accuracy is more important in large scale applications. When computing modern problems of long distance wave propagations and moderately high frequency propagations, there are great interests to develop time and spatial high-order and energy-preserving schemes.

Finite difference time domain method (FDTD) for Maxwell's equations was first introduced by Yee [17] in 1966 which was further developed by other researchers [12, 14] to a very efficient numerical algorithm in computational electromagnetics. However, the FDTD method is only conditionally stable and has large computational costs. Papers [13, 19] proposed ADI-FDTD schemes for Maxwell's equations which are unconditionally stable and of second order accuracy. Papers [1, 2] proposed energy-conserved spatial second-order S-FDTD schemes for Maxwell's equations which are efficient. The schemes are energy conserved, unconditionally stable and non-dissipative. Papers [7, 8] developed energy-conserved spatial second-order S-FDTD schemes for metamaterial electromagnetics. Paper [9] developed a spatial fourth order energy-conserved S-FDTD schemes, EC-S-FDTD(1,4) and EC-S-FDTD(2,4), for Maxwell's equations, which are fourth order accurate in space.

In this paper, we develop an even-odd cycled energy-conserved splitting finite difference time domain scheme for solving Maxwell's equations in two dimensions, Even-Odd cycled 4th order EC-S-FDTD, shorten to the EO-4th-EC scheme, with fourth order accuracy in space and second order accuracy in time. For EO-4th-EC

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scheme, we apply the spatial fourth order difference operators to a two stage splitting technique for each time step. The scheme consists of odd and even time step where for the odd time step, electric field in y-direction E_y and the intermediate value of magnetic field H_z are computed in stage one, following that electric field in x-direction E_x and H_z are solved in stage two. For the even time step, E_x and intermediate value H_z are computed in stage one and E_y and H_z are computed in stage 2. In this scheme, the spatial fourth order difference operators are obtained by a linear combination of two central differences on with a spatial step and one with three spatial steps while the boundary node difference operators are carefully defined keeping in mind the energy conservation and fourth order accuracy in space. Another important feature is that the use of even-odd two cycles achieves high-order accuracy in time while only using two stages in EO-4th-EC. We analyze in detail the stability, dispersion and phase error for the scheme. We also prove the scheme to be energy conservative. To find the stability of the scheme, the equivalent expressions for the even and odd time steps are computed by eliminating the intermediate terms. The expressions further allow us to compute the growth factor for each time step and the scheme overall. The growth factors help to determine the dispersion relationships of our scheme at each time step. The paper further focuses on numerical tests of the scheme. The phase velocity error of the proposed scheme are compared and computed with those other schemes such as ADI-FDTD, CN, EC-S-FDTDI, EC-S-FDTDII and EC-S-FDTD (2,4). The energy conservation, accuracy errors and the divergence free approximations are computed and compared to other schemes as well. Overall, the proposed scheme is found to be unconditionally stable and non-dissipative. The scheme also conserves energy and has higher accuracy.

This high order scheme is finally used in applications of MMIC such as coplanar waveguides. Coplanar waveguide, CPW, is made of two parallel plates made of conducting material, such as copper or gold, that run with some dielectric materials in between. In the numerical experiments, two lumped ports are attached on top of a grounded CPW to excite the waveguide. It is shown that the electric wave produced from the lumped ports is strong in the metal and weakens as it travels through the dielectric material. The dielectric substrate is made thick enough that the EM wave dies out before it reaches the conductor at the bottom of the GCPW. We also analyze the wave propagation of an electric wave in a transition between a CPW and a rectangular waveguide. This transition consists of back to back CPW and rectangular waveguide made of linearized array of via holes. The transition is excited with a magnetic source at the center of the domain. As the magnetic wave moves outwards it changes its shape as it moves into a different material.

2. Model and Scheme

2.1. Maxwell's Equations in 2D. Consider the 2D transverse electric polarization case in a lossless medium and there is no source. We have

$$\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon}\frac{\partial H_z}{\partial y}, \tag{1}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\epsilon}\frac{\partial H_z}{\partial x}, \tag{2}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}), \tag{3}$$