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NUMERICAL SIMULATIONS FOR SHALLOW WATER FLOWS OVER ERODIBLE BEDS BY CENTRAL DG METHODS

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Abstract. In this paper, we investigate the shallow water flows over erodible beds by using a fully coupled mathematical model in two-dimensional space. This model combines the nonlinear shallow water equations, the sediment transport equation and the bed evolution equation. The purpose of this paper is to design a well-balanced and positivity-preserving scheme for this model. In order to achieve the well-balanced property, the coupled system is first reformulated as a new form by introducing an auxiliary variable. The central discontinuous Galerkin method is applied to discretize the model. By choosing the value of the auxiliary variable suitably, the scheme can exactly balance the flux gradients and source terms in the "still-water" case, and thus the well-balanced property of the proposed scheme can be proved. Moreover, the non-negativity of the volumetric sediment concentration in the sediment transport equation is maintained by choosing a suitable time step and using a positivity-preserving limiter. Numerical tests are presented to illustrate the validity of the proposed scheme.

Key words. Shallow water equation, sediment transport equation, bed evolution, central discontinuous Galerkin method, well-balanced and positivity-preserving scheme.

1. Introduction

The nonlinear shallow water (SW) equations over a fixed bed [17] are widely adopted to model free-surface flows in rivers, flood plains and coastal regions. However, the highly energetic flows over erodible beds may induce the sediment transport and the bed evolution. Therefore, the nonlinear SW equations over a fixed bed cannot accurately predict the motion of flows over erodible beds.

To overcome this issue, various hydraulics models have been presented to simulate the fluid flow and the sediment transport in the past decades. In 2002, Pritchard and Hogg [30] investigated the suspended sediment concentration by using the erosional and depositional models and reported the exact solution for the suspended sediment transport under one-dimensional (1D) dam-break flow. Cao et al. [3] investigated a 1D dam-break flow over mobile bed by considering the induced sediment transport and morphological evolution. In 2003, Fagherazzi and Sun [11] proposed a coupled model of the SW equations, the suspended sediment equation and the Exner equation in 1D case, which was used to simulate the initiation and evolution of transportational cyclic steps. In 2006, Simpson et al. [32] proposed a two-dimensional (2D) mathematical model based on the SW equations and empirical functions for bed friction, substrate erosion and deposition. This model is an extension of the 1D model in [3] and [11], and can be used to simulate the channel initiation and drainage basin evolution associated with overland flow and morphological changes induced by extreme events such as tsunami. In 2008, Abderrezzak et al. [1] proposed a 1D coupled model for dam-break waves over movable beds. This model is built on the shallow water equations, the Exner equation and a spatial lag equation. Based on the 1D model in [3], Yue et al. [39] developed a coupled

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mathematical model in 2D space, which comprises the shallow water equations and the empirical relationships for sediment exchange over erodible beds. In 2010, Xia et al. [35] presented a 2D model for predicting dam-break flows over mobile beds. In this model, they modified the shallow water equations, and thus the effects of sediment concentration and bed evolution can be considered during the flood wave propagation. These equations are combined with the non-equilibrium transport equations for graded sediments and the equation of bed evolution. Li et al. [24] proposed a fully coupled system to model the 2D SW equations with sediment mass conservation and bed topography evolution. In 2012, Hu et al. [13] employed a complete layer-averaged conservation laws, including the mass and momentum conservation equations for the watersediment mixture and mass conservation equations for sediment and bed material respectively. Besides the impact of the friction, the impacts of morphological change and water entrainment are also considered in their model. In 2015, Liu et al. [27] built a fully coupled system combining the 2D SW equations with friction terms and the 2D sediment transport equations for the total load and the morphological evolution equation. Besides the above works, many related works have also been proposed, see, e.g., [5, 6, 29, 34].

To solve the models related to the shallow water flows over erodible beds, many popular numerical schemes have been developed in the literature, such as the explicit finite difference scheme [1], the finite volume methods based on the Godunov-type scheme, the Roe-MUSCL scheme or the slope limited centred scheme [32, 35, 24, 13, 31], the Godunov-type central-upwind scheme [27] and the discontinuous Galerkin (DG) method [14, 33]. In [13] and [27], the authors investigated the well-balanced property of the numerical schemes. In [14], the authors considered the DG discretization of the evolution equation of the bed due to the transport of sediment, but they did not consider the shallow water equations and the volumetric sediment concentration. In [33], the authors investigated the DG discretization of the shallow water equations and the evolution of the bed, but they still did not consider the volumetric sediment concentration.

In this paper, we will consider the numerical simulation of shallow water flows over erodible beds, which are governed by a fully coupled 2D system of the shallow water equations, the volumetric sediment concentration and the bed evolution equation presented in [27]. The numerical method is based on the central DG method which is different from the DG method used in [14, 33]. The proposed method is still well-balanced and can maintain the non-negativity of the volumetric sediment concentration. For the purpose of numerics, the coupled system is first reformulated as a new form by introducing an auxiliary variable, which is easier to achieve the well-balanced property. The reformulation is similar to the one in [25] and is called as a "pre-balanced" form. In fact, the "pre-balanced" form in [25] is a special case when the auxiliary variable is zero in our reformulation.

In this paper, the central DG method, which is a variant of the DG method [9, 8, 28] and free of Riemann solvers, is applied to discretize the reformulated model. There are also other schemes which are free of Riemann solvers in the literature, such as the upwind central scheme [27]. Some numerical schemes with Riemann solvers use a bound or a approximation value of the maximum eigenvalue [16]. The central DG method is also one of popular high order numerical methods, which was originally presented by Liu and his collaborators [26]. A well-balanced central DG method coupling with the finite element method was employed to solve the 1D fully nonlinear weakly dispersive Green-Naghdi model over varying topography ([21]). A