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BOUNDARY ELEMENT METHOD WITH HIGH ORDER IMPEDANCE BOUNDARY CONDITIONS IN ELECTROMAGNETICS

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Abstract. In this paper, we study boundary element method with high order impedance boundary conditions (HOIBC) to solve Maxwell's equations. The unknowns are electric and magnetic currents **J** and **M**. We propose several formulations and study the existence and uniqueness of the solution. Then, we discretize these formulations with a finite element method based on Lagrange elements. We give numerical tests of the HOIBC solution.

Key words. Boundary element method, scattering problem, high order impedance boundary condition.

1. Introduction

Radar and antenna system designers are interested in the theoretical study of the scattering of electromagnetic waves. Interest in this topic has prompted intensive research in this area long time ago. However rigorous analysis was not performed until recently. The development of the computing technology improves modeling possibility and it increases the interest in the scattering problem of electromagnetic waves. The difficulties of numerical methods include the necessity of using a large number of unknowns in the description of high frequency electromagnetic fields. The scattering problem is being studied for conducting bodies and for a perfect conducting body covered by a complex layer. The complex layer is considered as a homogeneous surface, as a chiral surface or as a frequency selective surface. Presently, the frequency selective surface is important for design artificial coatings of antenna.

There are many methods for solving the Maxwell's equations in harmonic regime. The first method is the volume method. It locates their computations all over the volume internal and external objects. It uses a domain containing the obstacles bounded by an artificial border. It considers the physical characteristics of the media, in particular the effects of anisotropy, but it requires a large number of unknowns and the management of explicit boundary conditions. Another method is the discontinuous Galerkin method, [12] that we used to solve elasticity problem [13].

Here, we choose the method of moments. It places unknowns on the boundaries of the object and it takes into account the boundary conditions. It allows reducing the exterior problem to a system of integral equations defined on the surface of the obstacle and we calculate equivalent magnetic and electric currents **M** and **J** which produce the true scattered fields in the exterior region. However, they can only be applied to homogeneous bodies. Here, we choose this method to solve time-harmonic scattering problem for a coated body.

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In order to ensure a unique solution to this boundary value problem it is necessary to apply boundary condition. Generally, we add impedance boundary condition on the surface of the object where impedance operator is a constant. It is known as standard impedance boundary conditions or Leontovith condition. But this approximation does not depend on incident angle at all. In this paper, we deal with higher order impedance boundary conditions to take account incidence angle. Recently, the higher order impedance boundary conditions have been studied in [2, 3]. This list is not exhaustive. These conditions take into account the incident angle at each point of the surface and include derivatives of tangential components of the fields that are equivalent to transverse wave numbers. The authors give several numerical results for body of revolution.

Later, the higher order impedance boundary conditions is applied to study the scattering problem from a finite planar or curved infinitesimally thin frequency selective surface embedded in a dielectric layer [8, 4, 5, 6]. The author introduces differential operators to express higher order impedance boundary conditions.

The organization of this paper is as follows. In section 2, we present the physical model. Then, in section 3, we give the high order approximation of the impedance boundary condition. In section 4, we establish formulations and we study existence and uniqueness of the solutions. In section 5, we give a discretization of this formulation and in section 6 we give several numerical tests.

2. Mathematical model of physical problem

We consider the scattering problem of electromagnetic waves ($\mathbf{E}$, $\mathbf{H}$) by a perfect conducting body with a complex coating. We denote Ω an open domain in $\mathbb{R}^2$ with a Lipschitz-continuous boundary $\Gamma = \partial\Omega$, which can be equipped with an exterior unit normal vector field $\mathbf{n}$, (see Figure 1). Electromagnetic waves propagate in $\Omega^+ = \mathbb{R}^2 \backslash \overline{\Omega}$. We illuminate this system by incident electromagnetic waves. Scattering waves occur when incident waves bounce off an object in a variety of directions. The amount of scattering waves that take place depends on the wavelength of the incident waves and structure of the object. We determine total electromagnetic fields $(\mathbf{E}, \mathbf{H})$ in Ω^+ as:

$$\begin{cases} \mathbf{E} = \mathbf{E}^{inc} + \mathbf{E}^{sc}, \\ \mathbf{H} = \mathbf{H}^{inc} + \mathbf{H}^{sc}. \end{cases} \tag{1}$$

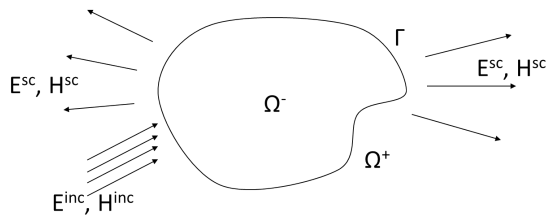

FIGURE 1. Scattering problem.

Superscripts *inc* and *sc* characterize incident and scattered fields, respectively. Waves propagation medium is described by two values ϵ (electrical permittivity)