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EFFICIENT GENERATION OF MEMBRANE AND SOLVENT TETRAHEDRAL MESHES FOR ION CHANNEL FINITE ELEMENT CALCULATION

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Abstract. A finite element solution of an ion channel dielectric continuum model such as Poisson-Boltzmann equation (PBE) and a system of Poisson-Nernst-Planck equations (PNP) requires tetrahedral meshes for an ion channel protein region, a membrane region, and an ionic solvent region as well as an interface fitted irregular tetrahedral mesh of a simulation box domain. However, generating these meshes is very difficult and highly technical due to the related three regions having very complex geometrical shapes. Currently, an ion channel mesh generation software package developed in Lu's research group is the only one available in the public domain. To significantly improve its mesh quality and computer performance, in this paper, new numerical schemes for generating membrane and solvent meshes are presented and implemented in Python, resulting in a new ion channel mesh generation software package. Numerical results are then reported to demonstrate the efficiency of the new numerical schemes and the quality of meshes generated by the new package for ion channel proteins with ion channel pores having different geometric complexities.

Key words. Finite element method, Poisson-Nernst-Planck equations, ion channel, membrane mesh generation, tetrahedral mesh.

1. Introduction

The Poisson-Boltzmann equation (PBE) [1, 2, 3, 4] and a system of Poisson-Nernst-Planck (PNP) equations [5, 6, 7] are two commonly-used dielectric continuum models for simulating an ion channel protein embedded in an membrane and immersed in an ionic solvent. While PBE is mainly used to calculate electrostatic solvation and binding free energies, PNP is an important tool for computing membrane potentials, ionic transport fluxes, conductances, and electric currents, etc. Both PBE and PNP have been solved approximately by typical numerical techniques such as finite difference, finite element, and boundary element methods. Among these techniques, finite element techniques can be more suitable to deal with the numerical difficulties caused by the complicated interface and boundary value conditions of PBE and PNP. Due to using unstructured tetrahedral meshes, they allow us to well retain the geometry shapes of protein, membrane, and solvent regions such that we can obtain a PBE/PNP numerical solution in a high degree of accuracy.

However, generating an irregular tetrahedral mesh for PBE/PNP finite element calculation can be very difficult and highly technical because a membrane region can cause a solvent region to have a very complicated geometrical shape, not mention that how to generate a membrane mesh remains a challenging research topic. In fact, because of the lack of membrane molecular structural data, generating a membrane mesh can become very difficult. To avoid this difficulty, the membrane

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can be simply treated as a piece of rectangular slab to separate a simulation box domain into the inner and outer solution regions. An ion channel protein region is then embedded in the slab to let ions flow across the membrane through the ion channel pore. Even so, the generation of a solvent mesh is still very difficult since a solvent region can have very complex interfaces with protein and membrane regions. To avoid this difficulty, a novel two-region approach is usually adopted to the development of an ion channel mesh generation scheme. That is, an ion channel simulation box is first divided into an ion channel protein region surrounded by an expanded solvent region without involving any membrane, where a mesh of this expanded solvent region is supposed to have been properly constructed such that it contains both membrane and solvent meshes that are to be constructed for an ion channel PBE/PNP finite element calculation; thus, the next work to be done is to develop a numerical scheme for extracting these two membrane and solvent meshes from the expanded solvent mesh. Based on this novel two-region approach, an ion channel mesh software package was developed in Lu's research group with more than seven-years efforts (2011 to 2018) [8, 9, 10, 11, 12, 13]. After four years, this package remains the unique one available in the public domain and applicable for PBE/PNP ion channel finite element calculation. For clarity, we will refer to it as ICMPv1 (i.e., Ion Channel Mesh Package version 1).

Clearly, the quality of membrane and solvent meshes extracting from an expanded solvent region strongly depends on the construction of a mesh extraction numerical scheme. In 2014 [9], cylinders (or spheres) were suggested to use in the separation of the membrane and pore regions but a separation process was mainly done manually. The first numerical extraction scheme was reported in 2015 [10], which significantly improved the usage and performance of ICMPv1. In this scheme, a walk-detect method was adapted to detect the inner surface of an ion channel pore numerically, making it possible for us to generate the solvent, membrane, and protein region meshes and an interface fitted mesh of a simulation box domain without involving any manual effort. However, the mesh quality and the performance of ICMPv1 rely on the selection of the walk step size, the number of searching layers, and the other mesh generation parameters. A proper selection of the values of these parameters turn out to be difficult and very time-consuming for an ion channel protein having an ion channel pore with a complicated geometrical shape.

Recently, ICMPv1 was adapted to the implementation of the new PBE/PNP ion channel finite element solvers developed in Xie's research group [6, 7, 14, 15]. During these applications, ICMPv1 was found to occasionally produce a membrane mesh that contains the tetrahedra belonging to a solvent mesh. It is possible to remove these false tetrahedra manually. For example, via a visualization tool (e.g., ParaView [16]), we may identify these false tetrahedra and then remove them by reconstructing a new mesh. We may also adjust the related parameters repeatedly until none of false tetrahedra occur in a membrane mesh, but doing so may be very time-consuming and may twist a protein, membrane, or solvent mesh due to using improper parameter values, causing the numerical accuracy of a PBE/PNP finite element solution to be reduced significantly. These cases motivated Xie's research group to develop more effective and more efficient numerical schemes than those used in ICMPv1. Eventually, the second version of ICMP, denoted by ICMPv2, has been developed by Xie's research group through a close collaboration with Lu's research group. The purpose of this paper is to present the new schemes