

FRACTIONAL ORDER LEARNING METHODS FOR NONLINEAR SYSTEM IDENTIFICATION BASED ON FUZZY NEURAL NETWORK

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Abstract. This paper focuses on neural network-based learning methods for identifying nonlinear dynamic systems. The Takagi-Sugeno (T-S) fuzzy model is introduced to represent nonlinear systems in a linear way. Fractional calculus is integrated to minimize the cost function, yielding a fractional-order learning algorithm that can derive optimal parameters in the T-S fuzzy model. The proposed algorithm is evaluated by comparing it with an integer-order method for identifying numerical nonlinear systems and a water quality system. Both evaluations demonstrate that the proposed algorithm can effectively reduce errors and improve model accuracy.

Key words. Fractional calculus, T-S fuzzy neural network, gradient descent method, nonlinear systems.

1. Introduction

System identification (SID) is indispensable both in traditional industry (chemical industry, refinery, etc.) and modern engineering fields, e.g., mechanical engineering [1], robotics and quadrotors [2], etc., since it can provide effective models for adaptive control and prediction [3–5]. Neural network (NN) based SID methods have aroused the interest of many researchers since the activation function in NN has multiple choices so that the NN could bear excellent performance in fitting nonlinear systems [6]. One of the widely used NN is the fuzzy neural network (FNN) [7, 8], which is a hybrid intelligent system based on fuzzy logic and neural network. By applying fuzzy sets and rules to the neural network, the FNN can handle fuzzy and uncertain information and has good generalization ability. The FNN has been widely applied in many fields, including electric power [9], machinery [10], and mathematics [11].

Takagi-Sugeno (T-S) fuzzy model is one of the most commonly used fuzzy neural network models which is proposed by Takagi and Sugeno in 1985. It describes the rule of 'if-then' and consists of two parts: an antecedent network and a consequent network. The antecedent network is used to match the antecedent of fuzzy rules, and the consequent network is used to generate the consequent of fuzzy rules. The total output is the weighted sum of the consequences of each fuzzy rule, where the weighting coefficient is the applicability of each rule. The parameters of membership functions can be initialized by the training samples, while associated parameters are identified by the recursive least squares algorithm [13]. Muralisankar made use of the Lyapunov-Krasovskii functional and stochastic analysis approach and established new delay-dependent stability criteria in terms of linear matrix inequalities (LMIs) in T-S fuzzy stochastic neural networks [14]. Li proposed a recognition method and simplified scheme for T-S fuzzy neural networks. The basic idea is that the structure identification of fuzzy neural networks is guided by the output approximation error attenuation in each cluster, with input space clustering

and sub clustering as the main steps [15]. Li studied observer based distributed time-varying delay T-S fuzzy neural network dissipation rate control, obtained an observer-based controller for the T-S fuzzy delayed model [16].

In addition, there are many identification algorithms for neural networks, such as the gradient descent (GD) algorithm [17] and the least squares (LS) algorithm [18]. In order to solve the problem of inefficient traditional lower and upper bound estimation (LUBE) model training, Liu adopted a new training scheme based on the GD method, which improved the LUBE model and enhanced efficiency [19]. Zheng focused on the most commonly used Stochastic Gradient Descent (SGD) algorithm in a mild decentralized setting and proposed a robust algorithm to handle unstable networks. [20].

Recently, fractional order-based SID algorithms have attracted many researchers [21, 22, 24]. Fractional differential calculus also has been a famous notion in mathematics for many years. It is an extension of traditional calculus and difference theory, and can describe many nonlinear and nonlocal phenomena. Some classical fractional derivatives include the Grunwald Letnikov derivative, Riemann-Liouville derivative, and Caputo derivative. Lupupa proposed a fractional order identification algorithm for wireless communication which had smaller errors than the conventional method because of its long-term memory ability and the reduction of model parameters [21]. Gehring analyzed fractional models from the perspective of mathematical algebra, unknown parameters and fractional order were identified solely from input-output signals, and further elastic materials were selected to illustrate the effectiveness of the method [22]. Liang investigated the input-output finite-time stability of fractional-order positive switched systems [23]. Aguilar used fractional calculus to reduce the number of parameters of the proposed neural network model, which simplified the complexity of the model and reduced the time required for digital simulation [24]. Compared with the integer order, fractional calculus takes into account the influence of the variables at the previous time and constructs a long memory function, so that the fractional order system can make use of the past information, and has a better effect on control and identification [25–27].

Taking into account the advantages of T-S fuzzy neural networks and fractional calculus, this paper proposes a fuzzy neural network for complex nonlinear system identification. The T-S fuzzy neural network is trained to fit nonlinear system observations in a linear way, and the parameters of the membership functions in the T-S fuzzy neural network can be estimated using a fractional-order gradient descent method. The main contributions of this paper are as follows:

- T-S fuzzy model is combined with the neural network to analyze complex nonlinear problems in a linear way. A fractional-order gradient descent learning algorithm is proposed to deal with the T-S fuzzy neural network.
- Evaluation is performed by comparing with integer order method on the identification of numerical nonlinear system and a water quality system, both of which show that the proposed algorithm can effectively reduce the error of the results and improve the accuracy of the model.

The rest of this paper is organized as follows. In Section 2, a T-S fuzzy model is constructed. In Section 3, the fractional order gradient descent updating rule is proposed to optimize the weights in the T-S fuzzy neural network, and a detailed pseudo code is presented. In Section 4, examples of numerical nonlinear model and water quality system are provided to verify the proposed algorithm. Finally, the evaluation of this work is reviewed and open issues are discussed for future research in Section 5.