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MAXWELL SOLUTIONS IN MEDIA WITH MULTIPLE RANDOM INTERFACES

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Abstract. A hybrid operator splitting method is developed for computations of two-dimensional transverse magnetic Maxwell equations in media with multiple random interfaces. By projecting the solutions into the random space using the polynomial chaos (PC) projection method, the deterministic and random parts of the solutions are solved separately.
There are two independent stages in the algorithm: the Yee scheme with domain decomposition implemented on a staggered grid for the deterministic part and the Monte Carlo sampling in the post-processing stage. These two stages of the algorithm are subject of computational studies. A parallel implementation is proposed for which the computational cost grows linearly with the number of random interfaces. Output statistics of Maxwell solutions are obtained including means, variance and time evolution of cumulative distribution functions (CDF). The computational results are presented for several configurations of domains with random interfaces.
The novelty of this article lies in using level set functions to characterize the random interfaces and, under reasonable assumptions on the random interfaces (see Figure 1), the dimensionality issue from the PC expansions is resolved (see Sections 1.1.2 and 1.2).

Key words. Maxwell Equations, Evolution of probability distribution, Monte Carlo simulation, Stochastic partial differential equation, random media, random interface, Polynomial chaos.

1. Introduction

Time evolution of waves in random media has important applications in a wide range of areas such as medical imaging, wave scattering, radar detection, ionospheric plasmas and photonics devices (see e.g. [11]). Although the problem under consideration here is a forward problem, our approach reveals the effects of random inputs and provide some insights on inverse problems, e.g. reconstruction of the interior of a human body from MRI or Ultrasound, recovery of interior structural parameters of machines from non-destructive measurements, ionospheric dynamics and related problems.

In this article, we study the evolution of the cumulative distribution functions (CDF) in time of electromagnetic (EM) fields. The randomness of the EM fields is inherited from the randomness of the locations of interfaces, i.e., it is uncertain where two or more different media interface (e.g. [4], [6], [9], [25]). In particular, the permeability and permittivity fluctuate randomly in space (independently of time) around their mean values. The EM fields with a single interface were studied and simulated in [12]. It has been demonstrated that the polynomial chaos expansion (PCE) methods are superior to Monte-Carlo methods in a number of applications (see [1], [2], [7], [8], [10], [17], [18], [27], [28], [29], [30], [32], [22]). Here, we extend the single interface to two or multiple interfaces which are described as level sets, $\{z(x,y) = \xi_i\}$ where the level function $z = z(x,y)$ is a function of x, y, and ξ_i are

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independent random variables or parameters (see Figure1). The case where the level function z depends on some random variables will be considered elsewhere. For simplicity, here we consider only a deterministic level function.

We note that conventional PCE methods have some severe limitations. If the number of random variables increases, the computational cost will grow exponentially as indicated in the polynomial chaos expansions (1.2), (1.10) and (1.19) below where the polynomials pertaining to each random variable are multiplied in a tensor product form. Thus the number of unknown coefficients (PC modes) grows exponentially and the computations are very expensive. Monte-Carlo methods are then more feasible. To avoid this curse of dimensionality, in this article, along with the time explicit scheme we will update the PC modes in each interval of the level $z = z(x, y)$ (see Figure 1, and also e.g. (1.18) for two random parameters). The computational cost then grows linearly as explained in Sections 1.1.2 and 1.2 below. See also the Conclusion below. This substantially reduces the computational time with parallel computing as demonstrated in Figure 5 below.

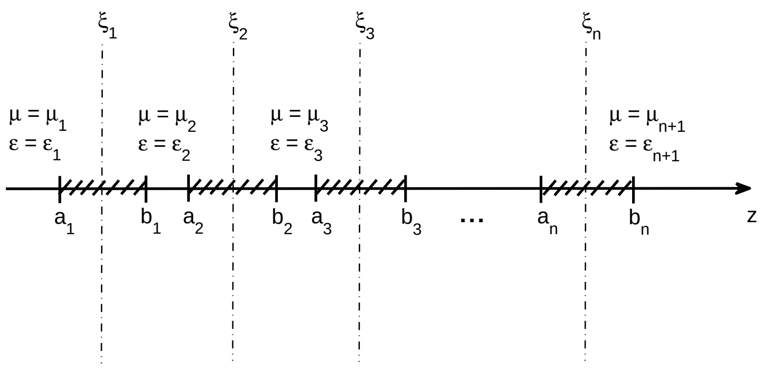

FIGURE 1. Multiple random interfaces $\{z(x, y) = \xi_i\}$ described by a level set function $z(x, y)$ where ξ_i are distributed over disjoint (a_i, b_i), $i = 1, 2, \cdots, n$.

We consider the following two-dimensional transverse magnetic Maxwell equations (e.g. [3], [19], [24]): for $(x, y, t) \in \mathbb{R}^2 \times (0, \infty)$,

$$
\begin{cases}
\dfrac{\partial H_1}{\partial t} = -\dfrac{1}{\mu}\dfrac{\partial E_3}{\partial y}, \\[2ex]
\dfrac{\partial H_2}{\partial t} = \dfrac{1}{\mu}\dfrac{\partial E_3}{\partial x}, \\[2ex]
\dfrac{\partial E_3}{\partial t} = \dfrac{1}{\epsilon}\dfrac{\partial H_2}{\partial x} - \dfrac{1}{\epsilon}\dfrac{\partial H_1}{\partial y}.
\end{cases}
\tag{1.1}
$$

The initial conditions are $H_1(x, y, 0) = h_1(x, y)$, $H_2(x, y, 0) = h_2(x, y)$, $E_3(x, y, 0) = e_3(x, y)$, where $H = (H_1, H_2, 0)^T$ is the magnetic field, $E = (0, 0, E_3)^T$ is the electric