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CHEBYSHEV SPECTRAL METHOD FOR UNSTEADY AXISYMMETRIC MIXED CONVECTION HEAT TRANSFER OF POWER LAW FLUID OVER A CYLINDER WITH VARIABLE TRANSPORT PROPERTIES

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Abstract. In this work, we study the unsteady axisymmetric mixed convection boundary layer flow and heat transfer of non-Newtonian power law fluid over a cylinder. Different from most classical works, the temperature dependent variable fluid viscosity and thermal conductivity are taken into account in highly coupled velocity and temperature fields. The motion of the fluid can be modeled by a time-dependent nonlinear parabolic system in cylindrical coordinates, which is solved numerically by using Chebyshev spectral method along with the strong stability preserving (SSP) third order Runge-Kutta time discretization. We apply the numerical solver to problems with different power law indices, viscosity parameter, thermal conductivity parameter and Richardson numbers, and compute up to the steady state. The numerical solver is checked by testing the spectral convergence of the numerical approximation to a smooth exact solution of the PDEs with source terms. Moreover, the combined effects of pertinent physical parameters on the flow and heat transfer characteristics are analyzed in detail.

Key words. Unsteady mixed convection, power law fluid, temperature dependent fluid viscosity, variable thermal conductivity, Chebyshev spectral method.

1. Introduction

Mixed convection (combined forced and free convection) flows are of interest and importance in a wide variety of fields such as petroleum, nuclear and environmental engineering. Flow over a slender cylinder is generally considered as axisymmetric instead of three dimensional problems, and the transverse curvature term contained in the governing equations strongly influences the velocity and temperature fields. Thus it has attracted great attention in the fields of steady/unsteady mixed convection flows under different situations [4, 13, 2, 12, 24].

One of the classical models of mixed convection flow is based on Newtonian fluid, which has been studied intensively. For steady flows, Kumari et al. [10] analyzed the effects of localized cooling/heating and suction/injection with a finite discontinuity on the mixed convection boundary layer flow in a thin vertical cylinder. Later, Mukhopadhyay [14] considered the axisymmetric mixed convection flow towards a stretching cylinder embedded in porous medium. Subsequently, Kaya [9] extended the problem to the case when the porosity of the porous medium is high, and non-similar solutions were obtained by the Keller-box method. For unsteady flows, unsteady mixed convection flow over a rotating vertical slender cylinder has been studied in [21]. Recently, Patil et al. [16] obtained a non-similar solution of an unsteady mixed convection boundary layer flow over a non-linearly stretching vertical slender cylinder, where the slender cylinder velocity varies arbitrarily with time.

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Besides the above, there are also some works on nanofluids. Grosan et al. [6] investigated the axisymmetric mixed convection flow past a thin vertical cylinder placed in a water-based copper (Cu) nanofluid. Moreover, Rashad et al. [20] studied the mixed convection boundary layer flow over a horizontal cylinder filled with a nanofluid, where they incorporated the Brownian motion and thermophoresis.

Another model is based on non-Newtonian fluid, which is a very commonly used model for gelled propellant fluid in aerospace application, foam fluid in petroleum industry, blood fluid in hemodynamic et al. [26, 25, 15]. However, there are only a few papers in the literature that deal with mixed convection flow of non-Newtonian fluid, and most works are on the inflow in the radial direction. For example, mixed convection from a heated semi-circular cylinder to power-law fluids in the steady flow regime is studied in [3, 1], and the unsteady flow was considered by Patnana et al. [17].

In all the above mentioned works, the thermo-physical properties of the fluid were assumed to be constant. However, these physical properties, especially fluid viscosity and thermal conductivity are affected by temperature [7, 11, 18, 19, 22, 23]. To the best of the authors' knowledge, little research was conducted for considering variable thermo-physical properties to boundary layer flows over a slender cylinder, especially the studies on unsteady flow of non-Newtonian fluid. In this paper, we focus on the effects of variable transport properties on unsteady mixed convection heat transfer of power law fluid.

Motivated by the above mentioned works, we perform our work in three aspects. First, we study the unsteady axisymmetric flow of a non-Newtonian power law fluid past a slender cylinder. Second, we consider the temperature dependent fluid properties. Third, we focus on developing numerical solutions with high-accuracy obtained by the Chebyshev spectral method [8], which can be sped up by the technique of Fast Fourier Transform (FFT).

2. Mathematical formulation

In this section, we construct the parabolic system to be solved numerically. Consider the unsteady axisymmetric mixed convection boundary layer flow over a vertical permeable slender cylinder of radius R placed in a non-Newtonian power law fluid with power law index n. Let t be the time variable and (u, v) be the velocity field along the (x, r) directions, where x and r are the axial and radial coordinates, respectively. By using the cylindrical coordinates, the domain is $(x, r) \in [0, +\infty) \times [R, +\infty)$. The density, velocity and temperature of the free flow at a remote distance from the cylinder are given by ρ_∞, U_∞ and T_∞, while the temperature of the static cylinder is T_W. The volumetric coefficient of thermal expansion is given as β_0, and the specific heat at constant temperature is assumed to be C_p. Under these assumptions, the governing equations for the mixed convection boundary layer equations are written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0, \tag{1}$$

$$\rho_\infty \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} \right) = \frac{1}{r}\frac{\partial}{\partial r}\left\{ r\frac{\mu_\infty}{[1+\gamma^*(T-T_\infty)]}\left|\frac{\partial u}{\partial r}\right|^{n-1}\frac{\partial u}{\partial r} \right\} + \rho_\infty g \beta_0 (T - T_\infty), \tag{2}$$

$$(\rho_\infty C_P)\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} \right) = \frac{1}{r}\frac{\partial}{\partial r}\left\{ k_\infty \left(1 + \varepsilon\frac{T-T_\infty}{T_W - T_\infty} \right) r\frac{\partial T}{\partial r} \right\}, \tag{3}$$

with the initial and boundary conditions given as follows.