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ON THE COMPARISON OF PROPERTIES OF RAYLEIGH WAVES IN ELASTIC AND VISCOELASTIC MEDIA

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Abstract. Dispersion properties of Rayleigh-type surface waves are widely used in environmental and engineering geophysics to image and characterize a shallow subsurface. In this paper, we numerically study the Rayleigh-type surface waves and their properties in 2D viscoelastic media. A finite difference method in a time-space domain is proposed, with an unsplit convolutional perfectly matched layer (C-PML) absorbing boundary condition. For two models that have analytical expressions of wave fields/dispersion curves, we calculate their wave fields and compare the analytical and numerical solutions to demonstrate the validity of this method. For the case where a medium has a high Poisson's ratio, say 0.49, traditional finite difference methods with a PML boundary condition are not stable when modeling Rayleigh waves but the proposed method is stable. For a laterally heterogeneous viscoelastic media model (Model 1) and a two-layer viscoelastic media model (Model 2) with a cavity, we use this method to obtain their corresponding Rayleigh waves. For several quality factors, the dispersion properties of these Rayleigh waves are analyzed. The results of Model 1 show that in a shallow subsurface, the phase velocity of a fundamental mode of the Rayleigh waves increases considerably with a quality factor Q decreasing; the phase velocity increases with Poisson's ratio increasing. The results of Model 2 indicate that the energy of higher modes of the Rayleigh waves become strong when Q decreases.

Key words. Rayleigh waves, elastic and viscoelastic media, convolutional perfectly matched layer, stability, finite difference method.

1. Introduction

In most surface seismic surveys, a different frequency component of a surface wave has a different phase velocity. This dispersion property is of fundamental interest in oil exploration, engineering and environmental studies. Rayleigh waves were used to construct S-wave velocity profiles [20, 24, 25, 27, 28], study attenuation [6, 26] and investigate cavities in a shallow subsurface [11].

The Rayleigh waves can be simulated by solving wave equations through numerical methods. One of the most popular numerical methods is the finite difference method (FDM). Several approaches were applied at a free surface to model these Rayleigh waves in elastic media using the FDM [12, 16, 19, 30, 29]. In particular, the accuracy of heterogeneous staggered-grid finite difference modeling of the Rayleigh waves has been studied by [4].

In reality, inelasticity of earth materials has an important influence on wave propagation, particularly on surface waves. It is necessary to simulate Rayleigh waves and analyze their dispersion properties in viscoelastic media, for example. Several works [5, 10, 9] have studied the Rayleigh waves in a viscoelastic half-space. Andersion *et al.* [1] gave a relationship between Rayleigh wave attenuation coefficients and the quality factors Q_P and Q_S for P- and S-waves. Xia [26] inverted a quality factor Q from Rayleigh waves using this relationship. However, this relationship is based on a layered earth model, and it is difficult to deal with complex media

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such as laterally heterogeneous media. The finite difference method may be used in the study of such cases. The approaches applied to handle a free surface boundary condition in viscoelastic media are similar to those in elastic media. Carcione [6] presented Rayleigh waves forward modeling in linear viscoelastic media. Hestholm [14] studied finite difference modeling of seismic scattering from free-surface topography in 3D viscoelastic media. Saenger and Bohlen [23] described the application of a rotated staggered grid (RSG) to viscoelastic wave equations. However, these works did not study the effect of a quality factor Q on the dispersion properties of Rayleigh waves.

Absorbing boundary conditions are used to suppress reflections from the truncated edges of a model in the FDM. Bérenger [2] developed an absorbing boundary condition called the perfectly matched layer (PML) to attenuate electromagnetic waves. This PML has been extended to absorbing acoustic and elastic waves [8, 13, 17]. Komatitsch and Martin [15] introduced an unsplit convolutional PML (C-PML) to improve the behavior of the classical PML at grazing incidence. However, the classical FDM with PML and C-PML is not stable in Rayleigh waves modeling with a high Poisson's ratio of media [31].

In this paper, we study the effect of a quality factor Q on the Rayleigh waves in order to better understand their dispersion properties. We propose a finite difference method to simulate the Rayleigh waves in viscoelastic media. This method uses the RSG proposed by [22], which has less numerical dispersion. The validity of the method is demonstrated using two models that have an analytic solution. The C-PML absorbing boundary condition is used in this method. It is stable to absorb the Rayleigh waves with a high Poisson's ratio of media. With our accurate modeling method, we study the dispersion properties of the Rayleigh waves with different values of the quality factor Q in a shallow subsurface. These Rayleigh waves are calculated in two models, a laterally heterogeneous model and a two-layer model with a cavity. The results show that the Q in the near-surface has a strong effect on the dispersion properties of the Rayleigh waves, and it needs to be considered in the analysis of the Rayleigh waves in the real world. Our method is based on a 2D finite difference method in a time-space domain, which can be extended in a straightforward way to the 3D case.

2. The Method

In this section we introduce the wave equations in viscoelastic media, a free boundary treatment, and an absorbing boundary condition. A finite difference method is then developed, and its validity and stability are tested.

2.1. Wave equations. We use a second-order displacement-stress form of the viscoelastic wave equations in 2D. In a time-space domain, the equations are given by [7]:

$$\rho \ddot{u}_x = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} + \rho f_x, \tag{1}$$

$$\rho \ddot{u}_z = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} + \rho f_z, \tag{2}$$

$$\sigma_{xx} = (\lambda_u + 2\mu_u)\frac{\partial u_x}{\partial x} + \lambda_u \frac{\partial u_z}{\partial z} + (\lambda_r + \mu_r)\sum_{l=1}^{L_1} e_{1l} + 2\mu_r \sum_{l=1}^{L_2} e_{2l}, \tag{3}$$