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FINITE VOLUME METHOD ON HYBRID MESHES FOR COASTAL OCEAN MODEL

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Abstract. In this paper we design a modified version of FVCOM by adopting hybrid meshes and shifting the placement of velocity variables from the centroids of elements to the middle points of edges. A simplified version of geostrophic equation is solved to test the new scheme, and illustrates a nearly uniform error distribution.

Key words. FVCOM, finite volume methods, hybrid meshes.

1. Introduction

The present study is a step toward the formulation of an unstructured-grid, primitive equation, 3D ocean model which features horizontal mixed triangular-quadrilateral meshes and vertically hybrid coordinate that is flexible in ocean applications with scales crossing river-estuary-shelf-basin-global. The model framework is partially adopted from the Finite Volume Coastal Ocean Model (FVCOM) which is an unstructured-triangular-grid open-source community ocean model and has been successfully applied to many scientific and engineering problems [2, 3, 4, 5].

The model development starts with the aim of modifying FVCOM to support a mixture of triangular and quadrilateral elements. Since at least two triangles are required to match the same area of a quadrilateral element, this modification allows the model domain be partitioned with an optimal number of non-overlapping elements for given grid resolution and could significantly relieve the computational cost of FVCOM, especially in high-resolution numerical modelling. Several potential benefits of using hybrid meshes in ocean modelling were also considered which includes but not limited to: 1) flexibility in either h (mesh size) or p (polynomial order) type of model refinement; 2) easily implementing velocity radiation boundary condition which may be a difficult task in an unstructured-triangular-grid model; 3) nesting an unstructured- and structured-grid model with the exactly matched boundary cells to reduce numerical instabilities due to interpolation between two meshes; 4) better numerical performance by reducing certain mesh shape related instability issues. These motivate us to examine the possibility of migrating the discretization method of FVCOM to the new hybrid mesh and try to understand its numerical properties and identify the potential aspects for future improvement.

In addition, the numerical design of FVCOM is based on finite volume methods in which the velocity variables (u, v) are placed at the centroid of each triangle

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while the pressure variables are at three vertices. This placement of variables simplifies the model by avoiding the specification of boundary conditions for velocities. However, it also brings some numerical difficulties which have to be treated with special attentions. First, without the velocity information along the boundary curves, the velocity gradient normal to the solid wall cannot be expressed explicitly. Therefore, in FVCOM, ghost cells are used to mimic the viscous boundary layer properly. Second, the velocities close to the open boundary can only be determined by the specified pressure conditions, which in most cases is the surface elevations caused by tidal fluctuations. To consider non-tidal components of velocities such as wind-driven flows or western boundary currents at open boundaries, one has to use mean-flow option in FVCOM, which actually is not very useful for real applications.

To avoid the above mentioned issues, the velocity variables in the new model are to be placed at the middle points of edges of the elements. It should be noted that this placement of variables is different from the lowest-order Raviart-Thomas element [9] as no constraint on the velocity is needed, while the latter requires the normal velocity be specified at edges and this is essentially the triangular C-grids that are widely used in unstructured-grid ocean modelling [1, 7]. Actually, our variable placement is very similar to the non-conforming linear elements of velocities that were used in a finite element scheme for two-layer shallow water equations [8]. Combining with linear conforming elements of surface elevations, the resulting discrete scheme is free of two-grid oscillations and appears suitable for coastal semi-enclosed basins circulation problems [8].

While the mesh is constructed in different way, and the placement of velocity and pressure variables changes, the numerical scheme of finite volume discretization changes accordingly. In this paper a FVCOM-like low order finite volume approach is described to resolve the velocities based on a mixed triangular-quadrilateral mesh. A simple geostrophic problem that determines the large-scale, steady-state ocean circulation is used to validate the model, discuss its numerical properties and identify the potential aspects for future improvement.

The remainder of this paper is organized as follows. In section 2, the numerical discretization of the geostrophic problem is introduced. In section 3, a numerical experiment is presented, followed by a detailed description of the results. The major finding and conclusions are summarized in section 4.

2. The new scheme with numerical treatments

Under the Boussinesq approximation, the incompressible, hydrostatic Navier-Stokes equation for the momentum in the model is

$$\frac{dU}{dt} + fk \times U = -g\nabla\eta - \frac{1}{\rho_0}\nabla B + \nabla \cdot A_h \nabla U + \partial_z A_v \partial_z U, \tag{1}$$

where $U = (u, v)$ represents the horizontal velocity components, ρ_0 is the reference density, B is the baroclinic pressure obtained through integrating the hydrostatic relation $\partial_z p = -g\rho$ from $z = 0$ with in-situ density ρ, g is the gravitational acceleration, η is the sea surface elevation, f is the Coriolis parameter, k is the vertical unit vector, A_h and A_v are lateral and vertical turbulent eddy viscosity respectively, and ∇ stands for 2D gradient or divergence operators.

The geostrophic equation is the lowest order of equation (1) for rapidly rotating fluids in the ocean interior with ignorable frictional effects and over large (> 100 km) spatial and long (> 2 days) temporal scales [6]. Recently, an unstructured grid, finite volume, 3D primitive equation, turbulent closure coastal ocean model is developed by Chen et al. [2, 3, 4, 5], which is called Finite Volume Coastal Ocean