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A CONVERGENCE ANALYSIS OF ORTHOGONAL SPLINE COLLOCATION FOR SOLVING TWO-POINT BOUNDARY VALUE PROBLEMS WITHOUT THE BOUNDARY SUBINTERVALS

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Abstract. We consider a new Hermite cubic orthogonal spline collocation (OSC) scheme to solve a two-point boundary value problem (TPBVP) with boundary subintervals excluded from the given interval. Such TPBVPs arise, for example, in the alternating direction implicit OSC solution of parabolic problems on arbitrary domains. The scheme involves transfer of the given Dirichlet boundary values to the end points of the interior interval. The convergence analysis shows that the scheme is of optimal fourth order accuracy in the maximum norm. Numerical results confirm the theoretical results.

Key words. Two-point boundary value problem, orthogonal spline collocation, optimal order of accuracy.

1. Introduction

The orthogonal spline collocation (OSC) technique is an efficient way to solve a wide variety of problems that are modeled by ordinary and partial differential equations, see [10] and references therein. OSC for solving a two-point boundary value problem (TPBVP) has been introduced and analyzed in [5]. We consider a new Hermite cubic OSC scheme to solve a TPBVP with boundary subintervals excluded from the given interval. Such TPBVPs arise, for example, in the alternating direction implicit (ADI) OSC solution of parabolic problems on some non-rectangular domains [3] with non-uniform consistent partitions. We expect to use the idea of transfer of Dirichlet boundary values presented in this paper to generalize the ADI OSC method of [3] to the solution of parabolic problems on arbitrary domains with uniform non-consistent partitions [4]. Figure 1 shows collocation points and horizontal line segments, without the boundary subintervals, on each of which a TPBVP is solved in the x-direction when the ADI OSC method is used to discretize a parabolic problem on an arbitrary domain with a uniform partition. Figure 2 shows the corresponding collocations points and vertical line segments, without the boundary subintervals, on each of which a TPBVP is solved in the y-direction. We believe that a theoretical convergence analysis of OSC for solving TPBVPs without the boundary subintervals is a first important and necessary step in justifying the ADI OSC method of [3] for solving parabolic problems on some non-rectangular domains as well as its generalization to arbitrary domains [4].

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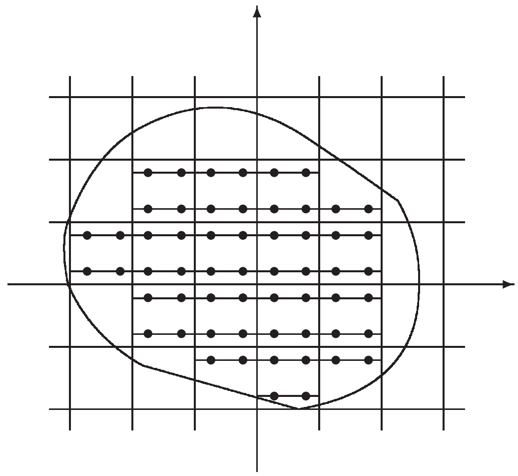

Figure 1. Horizontal line segments.

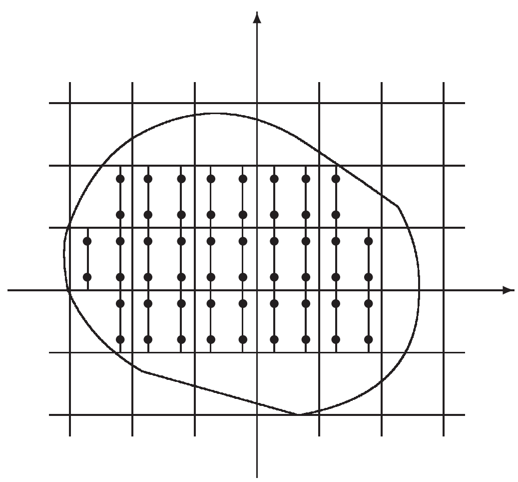

Figure 2. Vertical line segments.

A brief outline of the paper is as follows. In section 2 we present the formulation and implementation of the new Hermite cubic OSC method for the solution of a TPBVP with both end subintervals removed from the original interval. In particular, we explain the transfer of the Dirichlet boundary values on which our approach for higher dimensional problems will depend. In section 3, by converting the resulting linear system of equations to a tridiagonal one, we prove, using the discrete maximum principle, the optimal fourth order accuracy of our scheme. (For completeness we present the discrete maximum principle in the Appendix.) Numerical results presented in section 4 confirm our theoretical results. Concluding remarks are given in section 5.

2. OSC for two-point BVP without end subintervals

Consider the two-point BVP on $[a, b]$ with Dirichlet boundary conditions

$$Lu = f(x), \quad x \in (a, b), \quad u(a) = u_a, \quad u(b) = u_b, \tag{1}$$