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ASSESSMENT OF TWO APPROXIMATION METHODS FOR THE INVERSE PROBLEM OF ELECTROENCEPHALOGRAPHY

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Abstract. The goal of this paper is to compare two computational models for the inverse problem of electroencephalography: the localization of brain activity from measurements of the electric potential on the surface of the head. The source current is modeled as a dipole whose localization and polarization has to be determined. Two methods are considered for solving the corresponding forward problems: the so called *subtraction approach* and *direct approach*. The former is based on subtracting a fundamental solution, which has the same singular character of the actual solution, and solving computationally the resulting non-singular problem. Instead, the latter consists in solving directly the problem with singular data by means of an adaptive process based on an a posteriori error estimator, which allows creating meshes appropriately refined around the singularity. A set of experimental tests for both, the forward and the inverse problems, are reported. The main conclusion of these tests is that the direct approach combined with adaptivity is preferable when the localization of the dipole is close to an interface between brain tissues with different conductivities.

Key words. Electroencephalography, dipole source, electrostatics, inverse problem.

1. Introduction

Electroencephalography (EEG) is a diagnostic procedure which measures the electrical activity of the brain, by means of electrodes placed on the scalp. This non-invasive technique can be used for localizing current sources in the human brain [13].

Electromagnetic cerebral activity is due to the motion of ions in the active regions of the brain. This movement generates the so called *impressed current* (or primary current) that in turn creates ohmic currents in the surrounding environment called *return currents*. We are interested in determining the impressed current.

The reconstruction of the position and of some physical characteristics of the current density that gives rise to the EEG measurements is called the inverse problem. For an accurate reconstruction of the primary current it is important to be able to model realistically tissue conductivity inhomogeneities.

Since the frequency spectrum for electrophysiological signals in EEG is below 1,000 Hz, often between 0.1 and 100 Hz, most theoretical works on biomedical applications such as [8, 10, 22, 14] use the *static approximation of the Maxwell equations* in which the time variation of both electric and magnetic fields are disregarded. The static model is not the only possible simplification of the Maxwell equations. Other models that can be taken into account are the *electro-quasistatic model*, in which the time variation of the magnetic induction is not considered and the *magneto-quasistatic model* or *eddy current equations*, which are derived from the Maxwell equations by neglecting the time derivative of the electric field. It is also possible to study the problem using the *full system of Maxwell equations*. Some references on these approaches are [2, 1, 4, 11].

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We focus on the static model which leads to the electrostatics problem. We consider two strategies to approximate the potential for the electrostatics problem by using FEM. One of them is the "subtraction approach" which has been studied in [21, 22, 12, 9, 6, 19, 16], for example. In this formulation it is necessary to assume that the dipole is located in a region with a homogeneous conductivity. Then, it is possible to consider a more regular unknown, namely, the difference between the total potential and the fundamental solution with constant conductivity, which allows us to overcome the difficulties arising from the singularity of the source. Let us remark that this is a frequently used finite element approach for the numerical modeling in EEG.

The other method is the "direct approach", in which the unknown is the total potential and the dipole source is incorporated directly in the weak formulation which is solved by a finite element method. These two approaches have been compared in [5] in terms of computational complexity and accuracy. More recently, the direct approach was further analyzed in [3], where an a posteriori error estimate and an adaptive scheme which allows improving its efficiency were also introduced.

In this paper we report some numerical computations in order to compare the two methods: the well-known subtraction approach and the direct approach with adaptivity. The former is usually less expensive in terms of computational cost, because its solution does not present singularities and, consequently, coarse uniform meshes can be used for its finite element solution. However, we show that this is not always the case. In particular, we use them for the approximation of the inverse problem when the conductivity has a jump across the interface between different tissues (we recall that this is the case in the real physiological framework). We study in particular the case of a dipolar source located close to the interface between two regions with different conductivities (which again is physiologically realistic). The reported tests show that, in such a case, the subtraction approach can suffer from severe instabilities, while the direct approach is fairly stable. The instability of the subtraction approach is evident even for a two-dimensional problem with a simple geometry, and it seems that can be cured only on very fine meshes. Therefore, the instability clearly becomes more important when the problem is set in three dimensions on a more complex geometrical situation, since in that case very fine meshes are significantly more difficult to handle.

The paper is organized as follows: in Section 2 we introduce the methods and the assumptions to obtain well-posed problems, we establish some a priori error estimates and, finally, we introduce the a posteriori error estimator for the direct approach. In Section 3, we analyze the performances of the subtraction method and the direct approach with adaptivity for the corresponding forward problem. In Section 4 we explain in detail how we solve the inverse problem. In Section 5 we focus on how we generate reliable measurements for the simulations. In Section 6 we report numerical results for the inverse problem and, finally, in Section 7 we draw some conclusions.

2. Two approximation methods

We start introducing the equations.

2.1. Continuous problem.

In almost all the studies concerning the neural generation of electromagnetic fields the static approximation of Maxwell equations is