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NUMERICAL OPTIMAL POLLUTION CONTROL SUBJECT TO THE CONVECTION-DIFFUSION TRANSPORT EQUATIONS

YUJING YUAN AND DONG LIANG*

Abstract. In this paper, we develop an optimal control approach of pollution and emission reduction subject to convection-diffusion transport equations. A linked simulation-optimization method has been proposed, based on solving the convection-diffusion transport equations and solving the optimization procedure. The governing equations of the convection-diffusion-reaction equations with pollution sources are discretized by the splitting improved upwind finite volume scheme while the constrained differential evolution (DE) algorithm is applied to solve the global optimization procedure. The advantage of the approach is the external linking of the numerical simulation and the optimization procedure, minimizing both the weighted deviation between simulated concentrations and the smallest allowable concentrations at observation sites and the emission reduction cost at the pollution sources at same time. Numerical tests first check the convergence of numerical methods. Numerical experiments then show the performance of the approach for solving the optimal control problems of pollution and cost of emission reduction. The developed optimal control approach is efficient and it can be applied to more complex problems in applications.

Key words. Optimal pollution control, convection-diffusion equation, emission reduction cost, improved-upwind FV method, splitting.

1. Introduction

There are noticeable achievements in the development of economy, but the environment is recently deteriorating. For controlling and improving serious pollution of environment, the emission reduction plays an important role in the practical pollutant governing and balanced development. The common problems encountered over the world are groundwater pollution and air pollution ([2, 4, 5, 10, 12]). Emission control is the essential method to improve and control pollution and to protect the environment.

However, it is difficult to design the discharge strategy until the sources are identified with respect to their locations and magnitudes. Over the years, some methodologies have been proposed for groundwater source identification, such as the non-linear least-squares method [1], the geo-statistical approach [2], the constrained robust least square approach [15]. Paper [10] considered simultaneous estimation of aquifer parameters and identification of unknown pollution sources. Paper [12] used the artificial neural network for considering simple and complex scenarios, where the results were promising even with large measurement errors. But, during the procedure of minimizing the pollution, there is of great interest and difficulty to consider the cost affection of the controlling and reducing pollution emission in the real applications. Thus, it is an important task to study and develop the optimal control approach to the global optimization of pollution by considering the reduction cost in the environmental control and management.

In this paper, we propose a new optimal pollution control by minimizing the difference between the simulated concentration and the best environment allowing

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*Corresponding author. *E-mail: dliang@mathstat.yorku.ca.*

concentration at observation sites and the cost of the emission reduction at the pollution source points. A linked simulation-optimization model has been developed, based on the two-dimensional convection-diffusion transport equations and the differential evolution (DE) optimization algorithm. The advantage of the approach is the external linking of numerical simulation and the optimization procedure. Different from [10], our optimization objective function includes two terms. While the first term aims at minimizing the weighted deviation between the simulated concentration and the best environment allowing concentration at observation sites over time, the second term makes the cost of emission reduction as small as possible. The proposed optimal pollution control model is subject to the convection-diffusion transport equations and two other kind constraints of state and control. In the numerical scheme for solving convection-diffusion transport equations, we propose to use the operator splitting scheme combining with the improved-upwind finite volume method. The two-dimensional problems are split into two one-dimensional problems at each time step, and the second-order improved-upwind finite volume method avoids nonphysical oscillation and obtains the high accuracy. The constrained differential evolution (DE) optimization algorithm is considered to solve the optimization procedure, which provides the advantages of its global solution solving feature, simplicity, powerful search capability, compact structure and high convergence. Numerical tests firstly show the second-order accuracy of the improved-upwind FV method. We then give numerical experiments of the optimal control problems of pollution. For an example without considering the emission reduction cost, numerical results are given for the cases with different levels of perturbation to observation data and different locations of the source points and observation sites. Two other examples are finally considered for the cases involved emission reduction costs with different cost functions, where it also considers different observation locations and different flow velocities. Numerical results show that the emission reduction rates can be found for the optimal pollution control and the pollution control depends on the choice of protected zones and also depends on the velocity of flow. The developed optimal control approach is efficient and it can be applied to more complex pollution control problems in applications.

This paper is organized as follows. In Section 2, we present the governing equations of the two-dimensional convection-diffusion transport problems with the local point sources and then propose the optimal pollution control model. In Section 3, the numerical schemes and the optimization algorithm are given. In Section 4, numerical experiments are taken and analyzed.

2. Formulation of Optimal Control Problems

2.1 The governing equations. We consider the pollution problems with a polluted region $\Omega \subset R^2$ and boundary Γ, where the pollutant is discharged through n_s outfalls (see Figure 1). The pollution of contaminant is governed by the two-dimensional convection diffusion reaction equations with point sources.

$$(1) \qquad \frac{\partial c}{\partial t} + \vec{v} \cdot \nabla c - \frac{\partial}{\partial x}(D_x \frac{\partial c}{\partial x}) + \frac{\partial}{\partial y}(D_y \frac{\partial c}{\partial y}) + Rc = \sum_{l=1}^{n_s} q_l(t) \delta(x - x_{s_l}, y - y_{s_l})$$

$$(x, y) \in [0, L_x] \times [0, L_y], t \in (0, T],$$

$$(2) \qquad c(x, y, 0) = c_0(x, y), \qquad (x, y) \in [0, L_x] \times [0, L_y],$$