INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 15, Number 1-2, Pages 48–73

LINEAR AND QUADRATIC IMMERSED FINITE ELEMENT METHODS FOR THE MULTI-LAYER POROUS WALL MODEL FOR CORONARY DRUG-ELUTING STENTS

HUILI ZHANG[1], TAO LIN[2], AND YANPING LIN[3]

Abstract. In this paper, we consider a multi-layer porous wall model for coronary drug-eluting stents that leads to an interface problem whose coefficients have multiple discontinuous points, and an imperfect contact interface jump condition is imposed at the first discontinuous point where the stent meets the artery. The existence and uniqueness of the solution to the related weak problem are established. A linear and a quadratic immersed finite element (IFE) methods are developed for solving this interface problem. Error estimation is carried out to show that the proposed IFE methods converge optimally. Numerical examples are presented to demonstrate features of these IFE methods.

Key words. Linear immersed interface method, quadratic immersed interface method, multi-layer porous wall model, coronary drug-eluting stents, imperfect contact interface point.

1. Introduction

It is known that alteration of blood flow due to the narrowing or occlusion of an artery is one of the most common occurrences in cardiovascular diseases. A treatment for cardiovascular diseases is alteration of blood flow in which, in order to hold open and to provide structural stability to the damaged vessel, a drug-eluting stent (DES) is inserted in the artery. A number of mathematical models [40, 41, 44] are proposed to simulate the drug transfer in the arterial wall in this kind of treatment. As is well known that the arterial wall consists of many layers with different structural and chemical properties [23]. It is believed that a better modeling of the wall structure brings us a more effective description of the drug release from a DES. One of these complete wall models is the multi-layer wall model that takes into account the heterogeneous properties of the different layers constituting the arterial wall. Because the mass dynamics mainly occurs along the direction normal to the stent's coating, G. Pontrelli and F. Monte proposed a simplified one-dimensional (1D) multi-layer porous wall model in [42], see the illustration in Fig. 1 which is based on Fig. 2 in [42].

First of all, let us review this model briefly. In a general 1D framework, we consider a set of intervals $[\alpha_{i-1}, \alpha_i]$, $i = 0, 1, 2, \ldots, n$, having thickness $l_i = \alpha_i - \alpha_{i-1}$ modeling the drug coating ($i = 0$) and the arterial wall layers ($i = 1, 2, \ldots, n$), as shown in Fig. 1. At the initial time ($t = 0$), the drug is contained only in the coating and it is distributed with maximum concentration u_0 and, subsequently, released into the arterial wall. Here, and throughout this paper, a mass volume-averaged concentration $u(x, t)$ is considered.

We know that the metallic strut is impermeable to the drug, so there is no mass flux passes through the boundary surface at $x = \alpha_{-1}$. Thus, the dynamics of the drug in the coating $[\alpha_{-1}, \alpha_0]$ should satisfy the following 1D diffusion equation and

Received by the editors January 3, 2017 and, in revised form, April 13, 2017.
2000 *Mathematics Subject Classification.* 35R35, 49J40, 60G40.

related boundary-initial conditions:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial}{\partial x}(-D_0 \dfrac{\partial u}{\partial x}) = 0,\ x \in [\alpha_{-1}, \alpha_0], \\ -D_0 \dfrac{\partial u}{\partial x} = 0,\ x = \alpha_{-1}, \\ u(x,0) = u_0, \end{cases}$$

where D_0 is the drug diffusivity, u_0 the concentration in the coating.

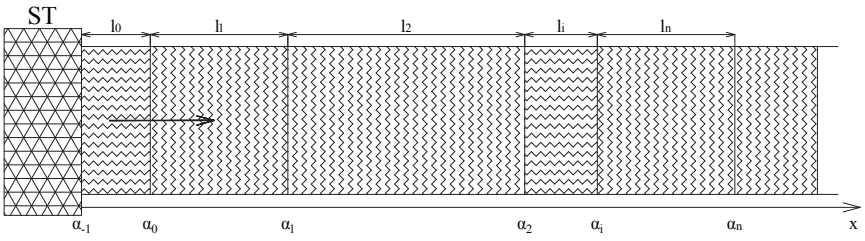

FIGURE 1. A sketch of the layered wall. ST indicates the metallic stent strut bearing the polymeric coating, while $[\alpha_{-1}, \alpha_0]$ means the polymeric coating. The continuous wall layers are defined by $[\alpha_{i-1}, \alpha_i]$, $i = 1, 2, \ldots, n$. This illustration is based on Fig. 2 in [42].

To prolong the drug release time, we need to slow down the drug release rate. To achieve this goal, a permeable membrane (called topcoat) of permeability p is placed at the interface ($x = \alpha_0$) between the coating and the arterial wall. Thus, the mass flux passed through it is continuous while the drug concentration might have a possible jump. In this case, the mass transfer through the topcoat can be described by the following second Kedem-Katchalsky equation:

$$\begin{cases} -D_0 \dfrac{\partial u(\alpha_0^-)}{\partial x} = p(\dfrac{u(\alpha_0^-)}{c_0} - \dfrac{u(\alpha_0^+)}{c_1}), \\ D_0 \dfrac{\partial u(\alpha_0^-)}{\partial x} = D_1 \dfrac{\partial u(\alpha_0^+)}{\partial x} - 2\delta_1 u(\alpha_0^+), \end{cases}$$

where, c_0 and c_1 are two constants relevant to the porosity. Hereafter, D_i is the diffusivity of drug and δ_i denotes for a constant characteristic convection parameter in $[\alpha_{i-1}, \alpha_i]$, $i = 1, 2, \ldots, n$.

Then, we consider the drug transfer in the layers of the arterial wall. In the i-th layer, the drug transfer obeys the following advection-diffusion-reaction equation and related initial conditions:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial}{\partial x}(-D_i \dfrac{\partial u}{\partial x} + 2\delta_i u) + \beta_i u = 0,\ x \in (\alpha_{i-1}, \alpha_i),\ i = 1, 2, \ldots, n, \\ u(x,0) = 0,\ x \in (\alpha_{i-1}, \alpha_i), \\ u(\alpha_n, t) = 0, \end{cases}$$