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MODELING AND SIMULATION OF THE ATMOSPHERIC DUST DYNAMIC: FRACTIONAL CALCULUS AND CLOUD COMPUTING

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This paper is dedicated to Professor Benyu Guo

Abstract. The dust aerosols have an important effect on the solar radiation in the Martial atmosphere and both surface and atmospheric heating rates, which are also basic drivers of atmospheric dynamics. Aerosols cause an attenuation of the solar radiation traversing the atmosphere and this attenuation is modeled by the Lambert-Beer-Bouguer law, where the aerosol optical thickness plays an important role. Through Angstrom law, the aerosol optical thickness can be approximated and this law allows to model attenuation of the solar radiation traversing the atmosphere by a fractional diffusion equation. The analytical solution is available in the case of one space dimension. When we extend the fractional diffusion equation to the case of two or more space variables, we need large and massive computations to approach numerically the solutions. In this case a suitable strategy is to use the cloud computing to carry out the simulations. We present an introduction to cloud computing applied to the fractional diffusion equation in one dimension.

Key words. Dust, solar radiation, fractional calculus, Mittag-Leffler type functions, fractional ordinary and partial differential equations, cloud computing, performance model, cost model

1. Introduction

Aerosols are minute particles suspended in the atmosphere which can scatter and absorb sunlight if they are sufficiently large. Thus, the aerosols interact both directly and indirectly with the Earth's radiation budget and climate. A direct effect of the interaction of the aerosols with the Sun radiation and climate is that the aerosols scatter sunlight directly back into space. By other hand, the aerosols in the lower atmosphere can modify indirectly the size of cloud particles, changing how the clouds reflect and absorb sunlight, thereby affecting the energy budget of the planet.

The dust particles are a type of aerosol with a significant effect on climate, because the dust is composed of particles of minerals that absorb sunlight as well as scatter it. By absorbing sunlight, the dust particles warm the layer of the atmosphere where they reside and this could inhibit the formation of storm clouds and expand the desert scenario.

In the particular case of the Martian atmosphere, the dust aerosols have an important effect on the solar radiation and both surface and atmospheric heating rates, which are also basic drivers of atmospheric dynamics. The importance of the dust in the martian exploration was recognized early in 1972 and since then dust is a target of atmospheric studies ([1], [2], [3]).

Under different Martian atmospheric scenarios, the measure of the amount of solar radiation at the Martian surface will be useful to gain some insight into the following issues:

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1) UV irradiation levels at the bottom of the Martian atmosphere to use them as an habitability index.
2) Incoming shortwave radiation and solar heating at the surface.
3) Relative local index of dust in the atmosphere.

Principally, the dust aerosols cause an attenuation of the solar radiation traversing the atmosphere. This attenuation is modeled by the Lambert-Beer-Bouguer law, where the aerosol optical thickness plays an important role and, through Angstrom law, the aerosol optical thickness can be approximated. However, the classical model often does not fit to the reality since the propagation of the solar radiation in the atmosphere is a complex process, whose dynamic is governed by different time/space scales. Thus, it is natural to think about integro-differential equations to describe a better modeling.

In this point, the Fractional Calculus offers new scenarios of modeling, since the fractional derivatives and integrals are non-local and they involve convolution kernels which act as memory factors. These properties make that the Fractional Calculus offers more suitable models to describe many physical phenomena, for instance, the dynamic of the martian atmosphere. Specifically, the attenuation of the solar radiation traversing the atmosphere can be modeled more accurately by a fractional diffusion equation, which provides a generalization of the classical Angstrom law.

This paper is organized as follow. In Section 2, we explain the fundamental laws that model the propagation of the solar radiation in a medium, and the basic elements of Fractional Calculus that we will use are introduced in Section 3. Next, in Section 4 the physics laws are reinterpreted in terms of Fractional Calculus and a new fractional model is obtained for the dynamic of the attenuation of the solar radiation traversing the martian atmosphere. In Section 5 we consider the numerical algorithms for one and two space dimensions. The associated simulations for one space dimension are described in section 6 and in the framework of the cloud computing. In this moment, we are applying the above computational strategy to the two dimensional case and the results will be published soon. Finally in section 6 we describe the cloud computing strategy and we implement it for the one dimensional fractional equation.

2. Foundations of the propagation of a radiation in a medium

2.1. Attenuation of the radiation. In the study of the propagation of the solar radiation in a medium, it is fundamental to know the attenuation of solar radiation traversing the atmosphere ([4], [5]). This attenuation is modeled by the Lambert-Beer-Bouguer law:

Definition 1. ***The Lambert-Beer-Bouguer law*** *establishes that the direct solar irradiance $F(\lambda)$ at the Mars's surface at wavelength λ is given by*

$$F(\lambda) = DF_0(\lambda)e^{-\tau(\lambda)m}, \tag{1}$$

where $F_0(\lambda)$ is the spectral irradiance at the top of the atmosphere, m is the absolute air mass, D is the correction factor for the earth-sun distance, and $\tau(\lambda)$ is the total optical thickness at wavelength λ.

2.2. Relevance of the aerosol optical thickness. The total optical thickness is obtained as the sum of the molecular scattering optical thickness $\tau_r(\lambda)$, the absorption optical thickness for atmospheric gases (O_2, O_3, H_2O, CO_2...) $\tau_g(\lambda)$, and the aerosol optical thickness $\tau_a(\lambda)$. In particular, $\tau_a(\lambda)$ can be obtained by direct solar spectral irradiance measurements by following the Angstrom Law [6]