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A NONOVERLAPPING DDM FOR GENERAL ELASTIC BODY-PLATE PROBLEMS DISCRETIZED BY THE P_1-NZT FEM

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Abstract. A nonoverlapping domain decomposition method (DDM) is proposed to solve general elastic body-plate problems, discretized by the P_1-NZT finite element method. It is proved in a subtle way that the convergence rate of the method is optimal (independent of the finite element mesh size), even for a regular family of finite element triangulations. This enables us to combine the method with adaptive techniques in practical applications. Some numerical results are included to illustrate the computational performance of the method.

Key words. Domain decomposition method, body-plate structure, new Zienkiewicz-type element (NZT), convergence analysis.

1. Introduction

Elastic multi-structures are composed of a number of substructures that have the same or different dimensions (e.g., bodies, plates, beams, etc.), coupled together with some junction conditions. They are widely used in the fields of aviation, aerospace, civil engineering, mechanical manufacturing, etc. In the past few decades, much work has been done about mathematical modeling, mathematical analysis and numerical solution for elastic multi-structure problems. We refer to [8, 11, 16, 17] and the references therein for details in this subject. Elastic multi-structures have a significant feature, that is, they are very complex from the global view point, but their substructures are quite simple comparably. Therefore, elastic multi-structure problems are particularly suitable for solutions through nonoverlapping domain decomposition methods (DDM). In [14], some substructuring method was proposed for solving the stiffened plate problem, based on conforming element discretization. In [15], two domain decomposition methods were given to solve a regular elastic body-plate problem. However, due to the use of finite element discretization in [27], the body and the plate members must have a cuboid shape, which greatly limits the applicability of these two methods.

In this article, we aim to propose and analyze a nonoverlapping DDM for solving a general elastic body-plate problem, discretized by the finite element method developed in [6], where P_1 conforming elements were used to discretize displacements on the body and longitudinal displacements on the plate, while the NZT element (cf. [28]) was used to discretize the transverse displacement. Hence, this method can apply to any bodies and plates with polyhedral/polygonal shapes. The ideas of constructing the related domain decomposition method are quite natural, similar to the second method in [15]. It can be viewed as a Dirichlet-Neumann type nonoverlapping method due to [19]. We refer the reader to the monograph [21] for a comprehensive understanding of this method and mention that it has been applied to solve a variety of coupling problems (cf. [12, 20, 30, 31]) with high efficiency.

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However, the convergence rate analysis of the proposed DDM is rather involved. We first introduce a Clément-type operator and then develop its error estimates in a subtle way. Then, we apply a new method to derive a certain spectral equivalence lemma. In light of these results, we are able to study the convergence rate of the DDM technically. It is proved that the convergence rate of the method is optimal (independent of the finite element mesh size), even for a regular family of finite element triangulations. It deserves to emphasize that there are few existing results about convergence rate analysis of nonoverlapping DDMs for regular finite element methods. Typically, it is often assumed that the restriction of the finite element triangulation to the interface should be quasi-uniform (cf. [5, 7]), to achieve the required spectral equivalence results (cf. [20, 21, 25, 29]). Hence, our convergence rate analysis developed here might be helpful in investigating convergence rates of some other nonoverlapping DDMs based on regular finite element discretizations. The other benefit is that it enable us to combine the nonverlapping DDM with adaptive techniques (cf. [5, 26]) to numerically solve the general elastic body-plate problem very effectively. Similar to the second method in [15], our method here only requires numerical solution of a pure body problem and a pure plate problem at each iteration step, which can be implemented by existing efficient numerical solvers. The relaxation parameter can be determined by numerical experience or by the power method. We provide an academic example to illustrate the computational performance of our method.

We end this section by introducing some notations and conventions frequently used later on. Throughout this paper, we adopt standard notations for Sobolev spaces [1, 5, 18], e.g., for a given open set G and a non-negative integer k, $H^k(G)$ consists of all $L^2(G)$-integrable functions whose weak derivatives with the total degree $\leq k$ are also $L^2(G)$-integrable, and the norm and seminorm are denoted by $\|\cdot\|_{k,G}$ and $|\cdot|_{k,G}$, respectively. $H_0^k(G)$ denotes the closure of $C_0^\infty(G)$ with respect to the norm $\|\cdot\|_{k,G}$, and the fractional-order Sobolev spaces are defined by real interpolation of Banach spaces. Moreover, denote by $P_k(G)$ the space of all polynomials over G with the total degree $\leq k$. We use the same index and summation conventions as described in [16, 17]. That means, Latin indices i, j, l take their values in the set $\{1, 2, 3\}$, while the capital Latin indices I, J, L take their values in the set $\{1, 2\}$. The summation is implied when a Latin index (or a capital Latin index) appears exactly twice. We also use the symbol "$\lesssim \cdots$" to denote "$\leq C \cdots$" with a generic positive constant C independent of the finite element mesh size and the functions under consideration, which may take different values in different appearances.

2. The P_1-NZT FEM for general elastic body-plate problems

As shown in Fig. 1, let (x_1, x_2, x_3) be a right-handed orthogonal system in the space $\mathbb{R}^3$, whose orthonormal basis vectors are denoted by $\{\boldsymbol{e}_i\}_{i=1}^3$, respectively. Let Ω be an elastic body-plate structure consisting of an elastic polyhedral body α and an elastic polygonal plate β (precisely speaking, β is the mid-surface of an elastic plate with thickness t_β), which is rigidly connected on the interface β_b, i.e.,

$$\boldsymbol{u}^\alpha = \boldsymbol{u}^\beta \quad \text{on } \beta_b, \tag{1}$$

where $\boldsymbol{u}^\alpha := u_i^\alpha \boldsymbol{e_i}$ and $\boldsymbol{u}^\beta := u_i^\beta \boldsymbol{e_i}$ are the displacement fields in α and β, respectively.