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A SPACE-TIME PARALLEL METHOD FOR THE OPTIC FLOW ESTIMATION IN LARGE DISPLACEMENTS AND VARYING ILLUMINATION CASE

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Abstract. We consider a unified variational PDEs model to solve the optic flow problem for large displacements and varying illumination. Although, the energy functional is nonconvex and severely nonlinear, we show that the model offers a well suited framework to extend the efficient methods we used for small displacements. In particular, we resort to an adaptive control of the diffusion and the illumination coefficients which allows us to preserve the edges and to obtain a sparse vector field. We develop a combined space-time parallel programming strategy based on a Schwarz domain decomposition method to speed up the computations and to handle high resolution images, and the *parareal* algorithm, to enhance the speedup and to achieve a lowest-energy local minimum. This full parallel method gives raise to several iterative schemes and allows us to obtain a good balance between several objectives, e.g. accuracy, cost reduction, time saving and achieving the "best" local minimum. We present several numerical simulations to validate the different algorithms and to compare their performances.

Key words. Optic flow estimation, large dislacements, variable illumination, adaptive finite elements, parallel and parareal computations, domain decomposition.

1. Introduction

The optic flow problem is a central research topic in computer vision since the last few decades. It is used in many fields, e.g. robotic for obstacle detection, in video compression and also in medical imaging. In the small displacements case, several approaches have been used, e.g. statistical methods, learning techniques, PDEs, etc ([2], [10], [27], [28]), see [22], [9] and the references therein. Despite the fact that these methods have been extended to the large displacements case, a lot of challenging questions remain, in both modelling and computational grounds. In fact, for small displacements (i.e. successive frames in the sequence are close), the constraint expressing the conservation of the pixel's intensity is usually linearized in a single step, which leads essentially to solve a linear system of PDEs. This is no longer true for large displacements and the problem requires, in the variational framework, to work with highly nonlinear and nonconvex energy functionals possessing possibly a lot of minima. Minimizing such functionals inevitably raises many difficulties (existence of -local- minimizers, regularity of solutions, well suitable optimization algorithms, cost of computations, etc). Moreover, phenomena like occlusions, change in the illumination within the scene, etc, should prevent from using techniques with too much smoothing effects (the solutions develop singularities which are important features to describe the motion).

In this article, we extend the approach considered in [18] for small displacements to the challenging problem of large displacements and varying illumination. It is a variational method which relies on the framework introduced in the seminal works of [24] and [20] and widely pursued by the community of mathematical image

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analysis, e.g. without being exhaustive [2, 8, 28], (see also [9] and the references therein). In this common framework, one minimizes an energy functional which consists of a data term that expresses the fundamental constraint of the optic flow estimation, namely the constancy of the pixel's intensity, and a regularization term among a family of well known regulaziers (e.g. Tychonov, TV, . . .). In all cases, this last part represents a diffusion term with prescribed diffusion coefficient/function.

In the article, we adopt the approach of control of the diffusion introduced in [6] which takes a spatially variable diffusion function, adjusted locally and adaptively to decrease the amount of diffusion near the singularities (high gradients areas).

To take into account the varying illumination, several approaches were suggested, e.g. constancy of the magnitude of the gradient of the images [8]. Gennert et al [15] suggested a "law" for such variations which relaxes the constancy of the intensity. In this work, we extend this method to construct a varying illumination function from the simple initial "law" suggested in [15]. Using the control approach, we obtain both a diffusion and a change of illumination with sophisticated profiles, i.e. piecewise smooth with non trivial jump sets (those giving some relevant informations on occlusions areas, shadow, . . .). Mathematically speaking, the method is convergent, in the Γ-convergence sense, to (special) bounded variation functions solutions [7].

We show that variational methods and the finite element discretisation offer a powerful and complete framework to solve the optic flow problem in the large displacement case. In fact, this setting allows us to decrease significantly the cost of computation by using coarse elements in the homogeneous regions where the flow field is smooth and refined meshes near the edges. Thus, we distinguish the geometry of the vector field (the optic flow), hence the meshes associated to it, and the meshes associated to the images in the sequence (constrained by the resolution and generally very fine and isotropic even if the motion is not), so that even for high resolution images, we may end up with meshes with only few elements and a sparse optic flow.

In order to increase the efficiency of the method with respect to additional constraints like fast computations and the high resolution sequences, it is necessary to use advanced tools of high performance computing, particularly the parallel programming. In fact, we have two main objectives in using parallel programming approaches: on one hand the (usual) goal of obtaining a significant time saving, and, in another hand, the aim of handling high resolution images. In addition, as the problem is nonconvex, we aim at computing a lowest -energy minimum. This last objective motivate our use of a parallel in time method (parareal algorithm) which may be considered as a time multi-grid method [13], and allows us to reach such lowest-energy solution for a given accuracy. This fact may be proven in special and simple problems (as for a standard multi-grid algorithms) but in our case it is a numerical evidence rather than a rigorous proof which is still open question. Another reason for the use of the parareal approach in our problem is the ability to use a different numerical scheme at the fine time scale than the one used at the coarse grid, or even the use of different physics between the two scales, i.e. a simplified version of the system of equations at one level, . . . This gives a supplementary flexible way to discretize such complex systems.

We study and compare three parallel computing methods: an MPI parallel method, in space, using a Schwarz domain decomposition technique which is clearly