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DYNAMIC LOAD BALANCING FOR THE PARALLEL, ADAPTIVE, MULTIGRID SOLUTION OF IMPLICIT PHASE-FIELD SIMULATIONS

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Abstract. In this paper we assess the performance of a selection of load balancing strategies for a parallel, adaptive multigrid solver that has been developed for the implicit solution of phase-field problems. The strategies considered include a number of standard approaches and a new technique that we propose specifically for multigrid solvers. This technique takes account of the sequential nature of the grid correction used in multiplicative multilevel algorithms such as multigrid. The paper focuses on two phase-field example problems which model the rapid solidification of an undercooled binary alloy: using isothermal and non-isothermal models respectively. We undertake a systematic comparison of the different load-balancing strategies for a selection of different adaptive mesh scenarios. We conclude that the optimal choice of load-balancing strategy depends critically on the computation to communication ratio of the parallel multigrid solver, and that in the computation-dominated limit our proposed technique is typically the most effective of those considered.

Key words. Load-balancing, parallel computation, multigrid.

1. Introduction

In recent years parallel processing has been used extensively in solving partial differential equations (PDEs) arising from engineering applications. In the numerical solution of PDEs a standard approach is to introduce a grid and apply finite difference [1, 2] or finite element methods [3] to discretize the governing equations to generate a system of algebraic equations. The solution of this system of algebraic equations gives the value of the unknowns at each degree of freedom. On very fine meshes the number of unknowns may easily reach several hundreds of millions ([4, 5, 6]) and solving such system in a reasonable time is a significant challenge. To reduce this computational time parallel processing is employed. In parallel computing the unknowns are typically divided into groups (generally corresponding to the number of processors) and are assigned to processors. Then these processors solve the equations associated with the unknowns in their assigned group simultaneously, with interprocessor communication to ensure global solution.

The partition of the unknowns among processors invokes two main issues. Firstly, balancing the workload is important to parallel programs for performance reasons. In some cases the number of unknowns and/or their distribution may change over time, due to adaptive mesh refinement (AMR) for example [6, 7, 8, 9, 10]. In such situations the dynamic redistribution of the workload is necessary to maintain an equal load balance [6, 10, 11, 12]. The other main issue of importance is the communication between processors: ideally this should be minimized since it creates additional overhead. It is not uncommon that there is trade off between good load-balancing and minimized communication volume between processors. This trade off becomes more difficult with AMR as the mesh is no longer uniform [11, 12].

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Moreover, the use of geometric muiltigrid creates another layer of difficulty due to the multiplicative nature of the algorithm, which requires work to be undertaken on each grid in a sequential manner. Consequently a load balancing strategy that does not take into account work on each grid or communication between grids at each level is necessarily not optimum [13, 14].

In this paper we introduce an adaptive parallel multigrid solver for a parabolic system in two or three spatial dimensions. We then consider different dynamic load balancing strategies and assess their impact on the performance of the solver for two different phase-field models arising from solidification problems. Phase field models of solidification are particularly demanding because they both track a moving boundary with a phase field, where a very fine mesh is required locally, and also evolve large scale diffusion fields such as temperature (where a much coarser mesh maybe sufficient). The solver that we use is described in detail in [6, 10]. It is based upon a cell centred finite difference scheme and fully implicit time stepping. We employ nonlinear FAS multigrid [15] as the solution scheme for the resulting nonlinear algebraic systems at each time step. The mesh generation and adaptivity are carried out by PARAMESH [14], a software library which builds a hierarchy of subgrids to cover the computational domain, with spatial resolution varying to satisfy the demands of the application. Our primary objective in this manuscript is to assess a selection of different load-balancing strategies applied to this family of phase-field problems. The approaches considered are Morton ordering, which is the default load-balancing approach used in PARAMESH [14], our own adaptation of Morton ordering for multigrid (described in [6]), as well as two standard approaches, which are recursive coordinate bisection (RCB) and graph partitioning, from the software package Zoltan [16, 17].

In the following section a brief overview of the software tools (our implicit non-linear multigrid solver, PARAMESH and Zoltan) will be given. Section 3 then discusses the dynamic load-balancing problem encountered in this work and briefly describes each of the load-balancing strategies considered in this study. The description of the time-dependent PDE problems that we solve and the background to the numerical experiments, are presented in Section 4. Finally we present our conclusions by reporting and discussing the results from a number of numerical experiments to compare these strategies when applied to a range of phase-field approximations with different characteristics.

2. Adaptive multigrid methods

The primary software tool used in this paper is called 'Campfire'. This package was developed by Goodyear et al ([6, 9, 18]) and contains several features. These features include spatial adaptivity, implicit and adaptive time stepping, dynamic load-balancing and a nonlinear geometric multigrid solver. The spatial adaptivity uses an external software library, PARAMESH [14].

PARAMESH generates meshes as the union of blocks of cells with different physical cell sizes, which are related to each other in a hierarchical fashion using a tree data structure. The blocks at the root of the tree have the physically largest cells, while their children have smaller cells and are said to be refined. Each child block is half as large as its parent block in each spatial dimension. The children of a block are nested so that they fit within their parent block and cannot overlap one another. In each block there are $N \times N \times N$ (N is a positive integer) cells arranged in a logically Cartesian fashion. Furthermore, blocks are wrapped by (typically) a