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COUPLING METHOD OF PLANE WAVE DG AND BOUNDARY ELEMENT FOR ELECTROMAGNETIC SCATTERING

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Abstract. In this paper we are concerned with the coupling of plane wave method and the boundary element method for electromagnetic scattering problems in unbounded domains, which are described by time-harmonic Maxwell's equations. We derive a coupled variational formula of the plane wave discontinuous Galerkin method and the boundary element method for the underlying model problem, and introduce a discretization of the coupled variational problem. The numerical results show that the proposed method is effective.

Key words. Maxwell's equations, plane wave DG method, boundary element method, coupling variational problem.

1. Introduction

Computational electromagnetic has been a hot research field for a long time due to its widely engineering application, such as electromagnetic analysis of circuits, antennas, and wireless communication systems, etc. Differential equations and integral equations are two basic forms for describing engineering problems. Therefore, numerical methods in computational electromagnetic can be classified according to equations they are based on. For example, the finite element method (FEM) and the finite difference time domain (FDTD) are based on differential equations, while method of moments (MoM) and its fast algorithms are based on integral equations. In recent years, with the enhancement of computer technique, many hybrid frameworks of different numerical methods in computational electromagnetic have been developed rapidly for the increasing requirement of more complicated engineering design and optimization.

Time-harmonic Maxwell's equations in unbounded domains is a basic model in the simulation of electromagnetic scattering. There are many methods for the numerical solution of this model problem, and among them the popular one is the coupling of finite element method and the boundary element method (see, for example, [1], [2], [3] and [4]). In recent years, the plane wave methods, which was first proposed for Helmholtz equation (see [5], [6] and [7]), have been extended to the discretization of time-harmonic Maxwell's equations in bounded domains (see [8], [9] and [10]), since the plane wave methods can generate higher accuracy approximations than the other methods for scattering problems with middle or high frequency.

In the present paper, we extend the plane wave method to the discretization of time-harmonic Maxwell's equations in unbounded domains. We first derive a coupled variational formula of the plane wave discontinuous Galerkin (PWDG) method and the boundary element method (BEM) for the electromagnetic scattering problems. Then we introduce a discretization of the coupled variational problem. A solution strategy for the resulting algebraic system is also proposed. In particular, to demonstrate the ability of the proposed method in dealing with complicated problems, we design a strategy for simplify of the coupled variational formula to the case that describes scattering problems of the composite dielectric and conducting objects. We apply the proposed method to simulate several

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electromagnetic scattering examples, and we find that the method is effective and can generate approximate solutions with higher accuracy than the coupling of the traditional finite element method and the boundary element method.

The paper is organized as follows: In Section 2, we describe the model problem; We give a variational formulation in bounded domains based on the PWDG method in Section 3; In Section 4, we derive a coupled variational problem of the PWDG method and the boundary element method; The discretization for the coupled variational problem is introduced in Section 5; In section 6, we discuss the application of the proposed method to a particular model describing the scattering problems of the composite dielectric and conducting objects; In Section 7, we report some numerical results; Finally, a conclusion is give in Section 8.

2. Description of underlying Maxwell equations

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain, with Lipschitz-continuous boundary $\Gamma = \partial\Omega$. Set $\Omega^c := \mathbb{R}^3 \backslash \Omega$. The relative permittivity and permeability for the domain Ω are denoted by ϵ_r and μ_r, while ϵ_r^c and μ_r^c are used to present the relative permittivity and permeability for the domain Ω^c. Here, the media in domain Ω^c is assumed to be homogenous dielectric, namely, ϵ_r^c and μ_r^c are constant real numbers. Let ϵ_0 and μ_0 be permittivity and permeability of free space. Then $\kappa_0 := \omega\sqrt{\epsilon_0\mu_0}$ is the wave number of the excitation in free space with $\omega > 0$ being the fixed angular frequency of the excitation. Moreover, the wave number of the excitation in domain Ω and Ω^c are $\hat{\kappa} := \kappa_0\sqrt{\epsilon_r\mu_r}$ and $\kappa := \kappa_0\sqrt{\epsilon_r^c\mu_r^c}$, respectively.

For a vector field $\mathbf{F}$ in Ω or Ω^c, define the traces on Γ by $\gamma_t\mathbf{F} = \mathbf{n} \times (\mathbf{F} \times \mathbf{n})$ and $\gamma_N\mathbf{F} = (\nabla \times \mathbf{F}) \times \mathbf{n}$, where $\mathbf{n}$ denotes the exterior unit normal vector on Γ from Ω into Ω^c.

Let $\mathbf{E}^c$ and $\mathbf{E}$ denote the complex amplitude of the scattered electric field in Ω^c and the total electric field inside Ω, respectively. Consider the transmission problem (cf. [1] and [3])

$$
(1) \qquad \begin{cases}
\nabla \times \nabla \times \mathbf{E}^c - \kappa^2\mathbf{E}^c = 0 & \text{in} \quad \Omega^c, \\
\nabla \times (\dfrac{1}{\mu_r}\nabla \times \mathbf{E}) - \kappa_0^2\epsilon_r\mathbf{E} = 0 & \text{in} \quad \Omega, \\
\gamma_t\mathbf{E}^c - \gamma_t\mathbf{E} = -\gamma_t\mathbf{E}_{inc}, \ \dfrac{1}{\mu_r^c}\gamma_N\mathbf{E}^c - \dfrac{1}{\mu_r}\gamma_N\mathbf{E} = -\dfrac{1}{\mu_r^c}\gamma_N\mathbf{E}_{inc} & \text{on} \quad \Gamma, \\
\lim\limits_{|\mathbf{x}|\to\infty}(\nabla \times \mathbf{E} \times \mathbf{x} - i\kappa|\mathbf{x}|\mathbf{E}) = 0,
\end{cases}
$$

where $\mathbf{E}_{inc}$ stands for the complex amplitude of the electric field associated with the incident wave.

We also consider an important variant of the above model. Let Ω be the union of two adjacent subdomains $\Omega^{(1)}$ and $\Omega^{(2)}$, as shown in Figure 1. The electric field $\mathbf{E}$ vanishes on $\Omega^{(2)}$ ($supp\ \mathbf{E} \subset \Omega^{(1)}$). This particular case describes scattering problems of the composite dielectric and conducting objects, such as micro-strip structures, antenna systems, aircraft or missile with radar radome, etc (refer to [11], [12] and [13]). The subdomain $\Omega^{(1)}$ corresponds to the dielectric region, while the conducting region, in which the electric field $\mathbf{E}$ vanishes, is denoted by the subdomain $\Omega^{(2)}$.

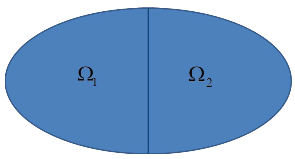

FIGURE 1. The schematic of the two subdomains $\Omega^{(1)}$ and $\Omega^{(2)}$.