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OPTIMIZATION FOR AUTOMATIC HISTORY MATCHING

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Abstract. History matching is an inverse problem of partial differential equation on mathematics. We adopt the constrained non-linear optimization to handle this problem, defining the objective function as the weighted square sum of differences between the wells simulation values and the corresponding observation values. We develop an optimization computing program that include Zoutendijk feasible direction methodQuasi-Newton method (BFGS) and improved Nelder-Mead simplex method, combined with a black-oil simulator, and discuss the convergence characters of algorithms in case studies about determining average porosity and directional permeability, determining low permeability strip between two wells and determining oil-water relative permeability curves.

Key Words. reservoirs numerical simulation, automatic history matching, inverse problem, optimization.

1. Problem

History matching is absolutely necessary for a real reservoir simulation, which is to find a suitable set of values for the simulator's input parameters such that the simulator correctly predicts the fluid outputs and the pressures of the wells on the reservoir. It is an inverse problem of partial differential equation on mathematics, and is not a well-posed problem [1-20]. Yet there must exist a solution reflecting real formation condition for a real reservoir problem. So we would focus attention on the stability of the history matching problem model and the algorithm feasibility, not to be concerned with the existence and singleness of the solution.

2. Mathematic Model

We adopt the constrained non-linear optimization most in use for inverse problem of partial differential equation to handle history matching problem, define the objective function as the weighted square sum of differences between the wells simulation values and the corresponding observation values:

$$f(X) = \sum_{i=1}^{n_w}\sum_{j=1}^{n_t}\sum_{k=1}^{n_k} \omega(i,j,k)[y^{obj}(i,j,k) - y^{cal}(i,j,k)]^2 \tag{1}$$

where y^{obj}, y^{cal} denote the observation values and simulator computing values respectively, ω denotes parameter scale coefficient *i, j, k* denote well number, time segment and data kind respectively, n_w, n_t, n_k are the maximum of *i, j, k* respectively , X denotes optimal vector.

For a general history matching problemthe objective function is an implicit function of the optimal vectorit needs to carrying out a simulation run to gain a objective function value, it is the uppermost computing cost. Therefore dealing equality constrained history matching problem, should adopt elimination method to reduce variable number, so as to optimization computing converge rapidly. So a general history problem can be posted as an inequality constrained nonlinear optimization problem

$$\begin{array}{lll} min & f(X) & X \in E^n \\ s.t & g_i(X) \geq 0 & i = 1, \cdots, m \end{array} \tag{2}$$

The optimal vector X, the objective function $f(X)$ and the inequality constrained function vector $G(X)$ are different for different history matching problem.

3. Algorithms

We develop an optimization computing program that include *Zoutendijk feasible direction methodQuasi-Newton method (BFGS) and improved Nelder-Mead simplex method* [21], combined with a black-oil simulator, and discuss the convergence characters of algorithms in some case studies.

Zoutendijk feasible direction method is a constrained nonlinear optimization method, it is in different ways to deal linear constraints and nonlinear constraints.

For linear inequality constraints optimization problem

$$\begin{array}{ll} min & f(X) \\ s.t & AX \geq b \end{array} \tag{3}$$

where, $f(\mathbf{X})$ is differential function, $\mathbf{A}$ is $m \times n$ matrix. $X \in E^n$, $\mathbf{b}$ is $\mathbf{m}$ dimension column vector. *Zoutendijk feasible direction method* transform determinating descent feasible direction $\mathbf{d}$ to solving following linear programming problem, according necessary conditions $\bigtriangledown f(X)^T d0, \quad A_1 d \geq 0$,

$$\begin{array}{ll} min & \bigtriangledown f(X)^T d \\ s.t & A_1 d \geq 0 \\ |d_j| \leq 1 & j = 1, \cdots n \end{array} \tag{4}$$

Linear search step restriction:

$$\lambda_{max} = \begin{cases} min\{\frac{B_j}{D_j} | D_j < 0\}, & D < 0 \\ \infty & D > 0 \end{cases} \tag{5}$$

where, $A_1\mathbf{X} = \mathbf{b}_1$, $A_2\mathbf{X} > \mathbf{b}_2$, $A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, $B = \mathbf{b}_2 - A_2\mathbf{X}_i, D = A_2\mathbf{d}_i$

For nonlinearinequality constraints optimization problem,

$$\begin{array}{lll} min & f(X) & \\ s.t & g_i(X) \geq 0 & i = 1, \cdots, m \end{array} \tag{6}$$

where$\mathbf{X} \in E^n, f(\mathbf{X}), g_i(\mathbf{X})$ are differentiable functions. *Zoutendijk feasible direction method* transform determinating descent feasible direction $\mathbf{d}$ to solving following *linear programming problem*, according necessary conditions $\bigtriangledown f(\mathbf{X})^T d < 0$, $\bigtriangledown g_i(\mathbf{X})^T d > 0$, $i \in I$, $I = \{i | g_i(X) = 0\}$

$$\begin{array}{lll} min & Z & \\ s.t & \bigtriangledown f(\mathbf{x})^T d - Z \leq 0 & \\ & \bigtriangledown g_i(\mathbf{x})^T d - Z \geq -g_i(\mathbf{x}), & i = 1, \cdots, m \\ & |d_j| \leq 1 & i = 1, \cdots, m \end{array} \tag{7}$$