INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 2, Supp , Pages 28–42

L^2-NORM ERROR BOUNDS OF CHARACTERISTICS COLLOCATION METHOD FOR COMPRESSIBLE MISCIBLE DISPLACEMENT IN POROUS MEDIA

NING MA, DANPING YANG AND TONGCHAO LU

Abstract. A nonlinear parabolic system is derived to describe compressible miscible displacement in a porous medium in non-periodic space. The concentration is treated by a characteristics collocation method, while the pressure is treated by a finite element collocation method. Optimal order estimates in L^2 is derived.

Key Words. compressible miscible displacement; characteristics line; collocation scheme; error estimate.

1. Introduction

The mathematical controlling model for compressible flow in porous media is given by

$$(1)\quad \begin{aligned} &(a)\ \ d(c)\frac{\partial p}{\partial t}+\nabla\cdot u=d(c)\frac{\partial p}{\partial t}-\nabla\cdot(a(c)\nabla p)=q, \qquad (x,y)\in\Omega,\ t\in(0,T] \\ &(b)\ \ \phi\frac{\partial c}{\partial t}+b(c)\frac{\partial p}{\partial t}+u\cdot\nabla c-\nabla\cdot(D\nabla c)=(\bar{c}-c)q, \quad (x,y)\in\Omega,\ t\in(0,T] \end{aligned}$$

where $\qquad c=c_1=1-c_2,\quad a(c)=a(x,y,c)=k(x,y)/\mu(c),$

$$b(c)=b(x,y,c)=\phi(x,y)c_1\{z_1-\sum_{j=1}^{2}z_jc_j\},\quad d(c)=d(x,y,c)=\phi(x,y)\sum_{j=1}^{2}z_jc_j.$$

c_i denote the concentration of the ith component of the fluid mixture, and z_i is the "constant compressibility" factor [1] for the ith component. The model is a nonlinear coupled system of two partial differential equations. Let $\Omega=(0,1)\times(0,1)$ with the boundary $\partial\Omega$, $p(x,y,t)$ is the pressure in the mixture,u is the Darcy velocity of the fluid, and $c(x,y,t)$ is the relative concentration of the injected fluid. $k(x,y)$ and $\phi(x,y)$ are the permeability and the porosity of porous media, $\mu(c)$ is the viscosity of fluid, $D(x,y)$ is molecular dissipation coefficient, q and $\bar{c}(t)$ etc. are just like the definition of [1,2].

We shall assume that no flow occurs across the boundary

$$(2)\quad \begin{aligned} &(a)\ \ u\cdot\nu=0 \quad on\ \partial\Omega, \\ &(b)\ \ D\nabla c\cdot\nu=0 \quad on\ \partial\Omega, \end{aligned}$$

Received by the editors January 1, 2004 and, in revised form, March 22, 2004.

2000 *Mathematics Subject Classification.* 65M25, 65M70.

The research was supported in part by Major State Basic Research Program of P. R. China grant G1999032803, and the Research Fund for Doctoral Program of High Education by China State Education Ministry.

where ν is the outer normal to $\partial\Omega$, and the initial conditions

$$(3) \qquad \begin{aligned} &(a) \quad p(x,y,0) = p_0(x,y), \quad (x,y) \in \Omega, \\ &(b) \quad c(x,y,0) = c_0(x,y), \quad (x,y) \in \Omega. \end{aligned}$$

The collocation methods are widely used for solving practice problems in engineering due to its easiness of implementation and high-order accuracy. But the most parts of mathematical theory focused on one-dimensional or two-dimensional constant coefficient problems [3-6]. In 1990's the collocation method of two-dimensional variable coefficients elliptic problems is given in [7].

The mathematical controlling model for compressible flow in porous media is strongly nonlinear coupling system of partial differential equations of two different types. Nonlinear terms introduce many difficulties for convergence analysis of algorithms. In the present article, we use different collocation technique to treat equations of different types, usual collocation method to solve the equation for pressure and characteristic collocation scheme to approximate the equation for concentration. We develop some technique to analyze convergence of collocation algorithm for this strongly nonlinear system and prove the optimal order L^2 error estimate. And we shall assume the coefficients $a(c), D(x,y), \phi(x,y), d(c), b(c)$ to be bounded above and below by positive constants independently of c as well as being smooth.

The organization of the rest of the paper is as follows. In Section 2, we will present the formulation of the characteristic collocation scheme for nonlinear system (1). In section 3, we will analyze convergent rate of the scheme defined in section 2. Throughout, the symbols K and ε will denote, respectively, a generic constant and a generic small positive constant.

2. Fully Discrete Characteristic Collocation Scheme

In this section, we will give some basic notations and definition for collocation methods, which will be used in this article. Then we will present the fully discrete characteristic collocation scheme for nonlinear system (1).

2.1. Notations and definition for collocation methods.

We make the partition of the domain Ω, which is quasi-uniform and equally spaced rectangular grid. The grid points are (x_i, y_j), $i = 0, 1 \cdots N_x; j = 0, 1 \cdots N_y$. Let

$$\delta_x : 0 = x_0 < x_1 < \cdots < x_{N_x} = 1, \qquad \delta_y : 0 = y_0 < y_1 < \cdots < y_{N_y} = 1$$

be the grid points along x-direction and y-direction respectively, and

$$h_x = x_i - x_{i-1}, \quad h_y = y_j - y_{j-1}, \quad h = \max\{h_x, h_y\}$$

be grid size along x-direction and y-direction and maximum size of partition respectively. Introduce the following notations:

$$\Omega_{ij} = (x_{i-1}, x_i) \times (y_{j-1}, y_j), \qquad I = [0,1]$$

$$I_x^i = [x_{i-1}, x_i], \quad I_y^j = [y_{j-1}, y_j],$$

for $i = 1, 2 \cdots N_x$ and $j = 1, 2 \cdots N_y$. Define function spaces as follows:

$$\mathcal{M}_1(3, \delta_x) = \{v \in C^1(I) |\ v \in P_3(I_x^i),\ i = 1 \cdots N_x\},$$

$$\mathcal{M}_1(3, \delta_y) = \{v \in C^1(I) |\ v \in P_3(I_y^j),\ j = 1 \cdots N_y\},$$

where P_3 denotes the set of polynomials of degree ≤ 3, and

$$\mathcal{M}_{1,P}(3, \delta_x) = \{v \in \mathcal{M}_1(3, \delta_x) :\ v(0) = v(1) = 0\},$$

$$\mathcal{M}_{1,P}(3, \delta_y) = \{v \in \mathcal{M}_1(3, \delta_y) :\ v(0) = v(1) = 0\},$$