INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 2, Supp , Pages 58–67

A PSEUDO FUNCTION APPROACH IN RESERVOIR SIMULATION

ZHANGXIN CHEN, GUANREN HUAN, AND BAOYAN LI

Abstract. In this paper we develop a pseudo function approach to obtain relative permeabilities for the numerical simulation of three-dimensional petroleum reservoirs. This approach follows the idea of an experimental approach and combines an analytical solution technique for two-phase flow with a numerical simulation technique for cross-sectional models of these three-dimensional reservoirs. The advantages of this pseudo function approach are that the heterogeneity of these reservoirs in the vertical direction and various forces such as capillary and gravitational forces can be taken into account in the derivation of the relative permeabilities. Moreover, this approach considers more physical and fluid factors and is more robust and accurate than the experimental approach. To reservoir engineers, the study of pseudo functions for the cross-sectional models of different types itself is the study of numerical simulation sensitivity of displacement processes in reservoirs. From this study they can understand the reservoir production mechanism and development indices.

Key Words. Reservoir simulation, pseudo function, mechanics of porous medium flow, cross-sectional model, non-dimensional cumulative production, relative permeability.

1. Introduction

The derivation of relative permeabilities in laboratory experiments [3] is carried out on core samples of porous media. The displacement mechanism in such samples is restricted to homogeneous cores. Moreover, in general, gravitational forces are ignored, and the magnitude of capillary forces is assumed to be very small. The relative permeabilities derived under such restricted conditions take into account only the microscopic heterogeneity of the porous media and viscous forces. If they were applied to the numerical simulation of a three-dimensional reservoir model, computational indices would be better than those observed in real situations. For a three-dimensional reservoir, the depth of each layer in the vertical direction is typically of the order of 10 m, and the permeability difference between different layers is of 10 times more. The heterogeneity in permeability can lead to the viscosity increase in a water-displacing-oil or gas-displacing-oil process; consequently, water or gas is produced at the very early stage from oil wells, and the amount of water or gas dramatically increases in these wells. Also, for such a reservoir, the density difference between the displacing fluid and displaced fluid often leads water and gas to the bottom and top of oil layers, respectively. Even for a homogeneous reservoir, the interface between different fluids can be non-homogeneous. In reality, capillary forces exist. The gravitational and capillary forces have very different influences on

Received by the editors September 23, 2004.
2000 *Mathematics Subject Classification.* 35K60, 35K65, 76S05, 76T05.

water and oil layers. The water layers can easily lead to the equilibrium of fluid motion in the vertical direction, and the layers with a lower water saturation can suck water from the layers with a higher water saturation under the influence of the capillary forces. But for the oil layers, the capillary forces offset the gravitational forces in those layers with a lower permeability, and this effect leads water in the higher permeability layers to the lower permeability layers. These two forces influence each other. This paper studys how to incorporate these complex forces (viscous, gravitational, and capillary) into the derivation of relative permeabilities for a three-dimensional reservoir. By reducing this reservoir to a two-dimensional cross-sectional reservoir and taking into account these forces in this reduced model, the relative permeabilities are obtained using the idea of the classical experimental approach and applied to the numerical simulation of the original three-dimensional reservoir. The computational development indices for this reservoir can accurately reflect various displacement mechanism factors in the study of numerical simulation sensitivity.

The difference between our pseudo function approach and other earlier approaches [4, 5, 6] lies in the fact that we combine pseudo functions with the sensitivity study by reservoir engineers and we derive these functions by combining analytical solution and numerical reservoir simulation techniques. The physical concepts in our approach is clear, its derivation is mathematically rigorous, and it is applicable to different reservoirs.

The rest of this paper is outlined as follows. In the next section we review the analytical solution technique. Then, in the third section we describe the derivation of relative permeabilities. In the fourth section we apply our pseudo function approach to a reservoir example. Finally, concluding remarks are given in the final section.

2. Analytical Solution of Two-Phase Flow

For a two-phase (e.g., water and oil) flow problem in a porous medium, Buckley and Leverett obtained an analytical solution in 1942 [1]. To combine the present pseudo function approach with an analytical solution approach, in this section we briefly review the derivation of this analytical solution.

2.1. Two-phase flow. For the flow of two incompressible, immiscible fluids in a porous medium, the mass balance equation for each of the fluid phases in the x-direction is

$$\phi\frac{\partial s_w}{\partial t}+\frac{\partial u_w}{\partial x}=0, \tag{2.1}$$

$$\phi\frac{\partial s_o}{\partial t}+\frac{\partial u_o}{\partial x}=0, \tag{2.2}$$

where w denotes the water phase, o indicates the oil phase, ϕ is the porosity of the medium, and s_α and u_α are, respectively, the saturation and volumetric velocity of the α-phase, $\alpha = w, o$. The volumetric velocities u_w and u_o are given by the Darcy law

$$u_w=-K\frac{K_{rw}(s_w)}{\mu_w}\frac{\partial p}{\partial x}, \tag{2.3}$$

$$u_o=-K\frac{K_{ro}(s_o)}{\mu_o}\frac{\partial p}{\partial x}, \tag{2.4}$$