INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 2, Supp , Pages 83–99

ROBIN TRANSMISSION CONDITIONS FOR OVERLAPPING ADDITIVE SCHWARZ METHOD APPLIED TO LINEAR ELLIPTIC PROBLEMS

HONGWEI LI AND JIACHANG SUN

Abstract. We consider overlapping Additive Schwarz Method(ASM) with Robin conditions as the transmission conditions(interior boundary conditions). The main difficulty left in this field is how to select the parameters for Robin conditions – these parameters have strong effect on the convergence rate of ASM. In this paper, we proposed the parameters for linear elliptic problems which seemed to be near optimal.

Key Words. domain decomposition, additive Schwarz methods, Robin transmission conditions.

1. Introduction

Classical additive Schwarz method(ASM) converges very slow for general problems. So, in most circumstances, this method can only be used as a preconditioner. On the other hand, ASM has high parallelism and is very suitable for coarse grain parallel computing. Many recent papers contribut to accelerating ASM. The technique is to replace the Dirichlet transmission conditions posed on the interfaces with some more general or exact conditions such as absorbing conditions, open conditions etc. The essence of these conditions is that they are more *exact* on the interfaces so that the corresponding ASM should converge faster. However, these conditions are always global coupled. So, in actual applications, these conditions should be localized by some kind of approximations. Taylor expansion was first used, and some other approximations were also introduced[6]. But it seems that these approximations hold only for simple problems that Fourier analysis can apply.

In this paper, the Dirichlet transmission conditions of the classical overlapping additive Schwarz method are replaced by Robin conditions directly. We hope that by selecting proper parameters for the Robin conditions, the corresponding ASM would converge more rapidly.

Robin transmission conditions were first introduced into domain decomposition by P.L.Lions in [9, 10, 11]. Since then, many papers followed.

Generalized Schwarz splitting method with Robin transmission conditions was proposed by Tang [12], which gave the initial impetus to our work in this field. Optimized Schwarz methods, proposed by M.J. Gander, L.Halpern and F.Nataf, try to get the optimal Robin parameters by Fourier analysis [6]. This idea was further utilized in [1, 8, 5, 7, 4].

Received by the editors January 1, 2004 and, in revised form, March 22, 2004.
2000 *Mathematics Subject Classification.* 65Mxx, 65Nxx, 65Yxx, 68Wxx.
This research was supported by the Basis Research Foundation (CXK25073) of Institute of Software, Chinese Academy of Sciences.

Absorbing conditions for domain decomposition methods have been analyzed by Zhao[2]. In that paper, Robin transmission conditions were analyzed by Taylor expansion.

Though many authors and papers have talked about Robin transmission conditions for additive Schwarz methods, the main difficulty – lacking of a simple and uniform way to choose good Robin parameters, is still remaining, even if for simple problems like Laplace equation.

This paper is motivated by generalized Schwarz splittings proposed by W.P. Tang and optimized Schwarz methods proposed by M.J. Gander. And we try to determine the optimal (or near optimal) Robin parameters for general linear elliptic problems.

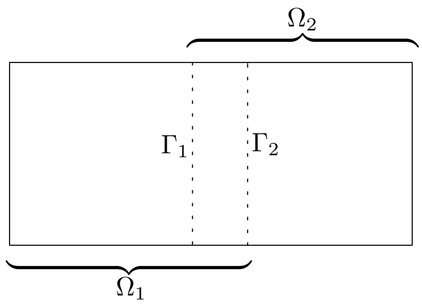
The key model problem for this paper is

$$-\Delta u + qu = f \quad (\Omega) \tag{1}$$

$$u = g \quad (\partial\Omega) \tag{2}$$

where $\Omega = (0,1)^d$, $d = 2, 3$, $q > 0$.

Suppose domain Ω is partitioned into two overlapping subdomains Ω_1 and Ω_2



Our aim is to derive the optimal(or near optimal) Robin parameters λ for the following additive Schwarz method (two subdomain case)

For any given initial values u^0, v^0, solve the following problems iteratively until convergence

$$-\Delta u^n + qu^n = f, \quad (\Omega_1) \tag{3}$$

$$\frac{\partial u^n}{\partial n} + \lambda u^n = \frac{\partial v^{n-1}}{\partial n} + \lambda v^{n-1} \quad (\Gamma_2) \tag{4}$$

$$-\Delta v^n + qv^n = f, \quad (\Omega_2) \tag{5}$$

$$\frac{\partial v^n}{\partial n} + \lambda v^n = \frac{\partial u^{n-1}}{\partial n} + \lambda u^{n-1} \quad (\Gamma_1) \tag{6}$$

where n denotes the outward normal direction of the subdomain under consideration. We will call above method as RASM(λ), so that it can be distinguished from ASM.

The main result of this paper is that for high dimensional model problems, the optimal Robin parameters can be determined as $\lambda_{opt} = \sqrt{q + (d-1)\pi^2}$, $d = 2, 3$.

2. Analysis for one dimensional Laplace equation

Suppose the domain $\Omega = (0,1)$, $0 < \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < ... < \alpha_{ns-1} < \beta_{ns-1} < 1$. $\Omega_1 = (0, \beta_1)$, $\Omega_2 = (\alpha_1, \beta_2)$, …, $\Omega_{ns-1} = (\alpha_{ns-2}, \beta_{ns-1})$,$\Omega_{ns} = (\alpha_{ns-1}, 1)$.