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A SLOPE LIMITING PROCEDURE IN DISCONTINUOUS GALERKIN FINITE ELEMENT METHOD FOR GASDYNAMICS APPLICATIONS

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Abstract. In this paper we demonstrate the performance of a slope limiting procedure combined with a discontinuous Galerkin (DG) finite element solver for 2D compressible Euler equations. The slope limiter can be categorized into van Albada type and is differentiable. This slope limiter is modified from a similar limiter used in finite volume solvers to suit the needs of the DG solver. The gradient in an element is limited using the weighted average of the face gradients. The face gradients are obtained from the area-weighted average of the gradient on both sides of the faces. The slope limiting process is very suitable for meshes discretized by triangle elements. The HLLC (Harten, Lax and van Leer) or the local Lax-Friedrich (LLF) flux functions is used to compute the interface fluxes in the DG formulation. The second order TVD Runge-Kutta scheme is employed for the time integration. The numerical examples including transonic, supersonic and hypersonic flows show that the current slope limiting process together with the DG solver is able to remove overshoots and undershoots around high gradient regions while preserving the high accuracy of the DG method. The convergence histories of all examples demonstrate that the limiting process does not stall convergence to steady state as many other slope limiters do.

Key Words. Discontinuous Galerkin (DG), slope limiting and 2D inviscid compressible flows.

1. Introduction

Discontinuous Galerkin (DG) method has been gaining popularity in computational fluid dynamics (CFD) in recent years [8, 9, 27]. Indeed, the DG method can be considered as a mixture of classic finite element method (FEM) and finite volume method (FVM). In the DG method, the advantageous features of the FEM and FVM are combined resulting in a robust and accurate numerical scheme for

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solution of problems involving shocks and other discontinuities. The variational form for each element is obtained by multiplying the governing equations with a test function and integrating conservative fluxes by parts. This results in boundary fluxes normal to the element interfaces. The inviscid flux is upwinded using an approximate Riemann solver.

Compared to the stabilized FEM, such as the streamline-upwinding/Petrov-Galerkin (SUPG) [1, 2, 3], the DG method is capable of sharper representation of the discontinuities in the solutions. In the DG method, the solution across each element can be discontinuous, therefore the DG method is naturally a better solution strategy for problems involving shocks and discontinuities. The DG method also eliminates the need for SUPG stabilization in advection dominant flows. In the DG method, the upwind fluxes provide the necessary stabilization. In addition, the *hp* refinement can easily be implemented in this method [9, 20] because hanging nodes are allowed in the DG method. The DG method is more compact than the FVM. In the finite volume method, the reconstruction within a cell relies on a cluster of neighboring cells using the path integral method or the least square method. If higher spatial accuracy order is desired in FVM, the number of supporting cells has to be increased. In contrast, in the DG method, linear or higher order interpolation functions can be employed to obtain the solutions at any points inside the element. The supporting elements are the same regardless of the spatial accuracy. This compactness makes the DG method more stable and easier to implement than the finite volume method. However, compared to the stabilized Galerkin finite element formulations, the DG methods require the solutions of systems of equations with more unknowns. However, if high order elements are used in the DG method, a very coarse mesh can be used to attain sufficient solution resolution [6]. Therefore the disadvantage of the DG method can be outweighed by its outstanding advantages.

It is well known that the nonphysical oscillations around high gradient discontinuities exist for linear stabilization techniques [1]. The oscillations are sometimes severe enough to cause stability problem. A discontinuity capturing [1] or an appropriate limiter is a common cure for this problem. Aliabadi and Tu [4] use discontinuity capturing method commonly used in SUPG/GLS finite element solvers as an alternative to slope limiters in their DG solver for 2D transient Euler equations. Sun and Takayama [22] employ a smoothing step in addition to the advection step to smooth the solution around high gradient regions. Their method is suitable for quadrilateral grids. The main defect of the methods mentioned above is that they usually require some user-defined parameters making them problem dependent. Many slope limiters used in the finite volume methods can be modified to meet the needs of the DG method. Slope limiters are usually parameter free. In [8], a limiter using maxmod functions is presented. Their limiter has the advantage over usual minmod based limiters [21], since it would not flatten smooth extrema. Hoteit et al. [17] introduce an extension of van Leer's slope limiter for two-dimensional DG method using unstructured quadrangular or triangular meshes. Unfortunately, there are two drawbacks with the use of slope limiters. One is the possible accuracy degradation in smooth regions; the other is that the convergence may be severely hampered. The main reason is that the limiters presented above are non-differentiable. It could be active in near uniform region. To overcome these drawbacks, some techniques have been adopted in practice. Venkatakrishnan [28] devised a new limiter that may accelerate the convergence rate. But it is not monotone. In addition, the constant in Venkatakrishnan's limiter is case independent.