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NUMERICAL INVESTIGATION OF KRYLOV SUBSPACE METHODS FOR SOLVING NON-SYMMETRIC SYSTEMS OF LINEAR EQUATIONS WITH DOMINANT SKEW-SYMMETRIC PART

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Abstract. Numerical investigation of BiCG and GMRES methods for solving non-symmetric linear equation systems with dominant skew-symmetric part has been presented. Numerical experiments were carried out for the linear system arising from a 5-point central difference approximation of the two dimensional convection-diffusion problem with different velocity coefficients and small parameter at the higher derivative. Behavior of BiCG and GMRES(10) has been compared for such kind of systems.

Key Words. convection-diffusion problem, central difference approximation, Krylov subspace methods, BiCG, GMRES(10), triangular preconditioners, non-symmetric systems, eigenvalue distribution of matrices

1. Introduction

The convection-diffusion equation is of much importance for modelling flow problems in computational fluid dynamics. While studying the property of a model convection-diffusion problem we can make some assumptions about the behavior of practical problems.

Let us consider the steady convection-diffusion problem:

$$(1)\qquad \begin{cases} -Pe^{-1}\Delta u + \frac{1}{2}\{v_1 u_x + v_2 u_y + (v_1 u)_x + (v_2 u)_y\} = f, \\ u(x,y)|_{\partial\Omega} = 0, \quad div\,(\bar{v}) = 0, \quad \bar{v} = \{v_1, v_2\}, \\ (x,y) \in \Omega = [0,1]\times[0,1], \quad f = f(x,y), \quad u = u(x,y) \end{cases}$$

where Pe is Peclet number, $\bar{v} = \{v_1, v_2\}$ is velocity vector. The first term in (1) describes the diffusion process while other terms correspond to the convective process. The magnitude of dimensionless parameter Pe determines the ratio of the convection process to the diffusion one. When Pe is greater than a certain constant and boundary conditions are in disagreement with the right-hand side there arise singular perturbation problems with boundary and interior layers [2].

The choice of discretization method for problem (1) and appropriate iterative method for the corresponding linear system is very important. There are various ways to discretize (1). In the context of finite difference, the most widespread schemes are the central difference (second-order scheme) and the upwind (firesorder scheme). It is well known [2] that in general, linear system with M-matrix [12] can be obtained by applying the upwind schemes while positive real matrix

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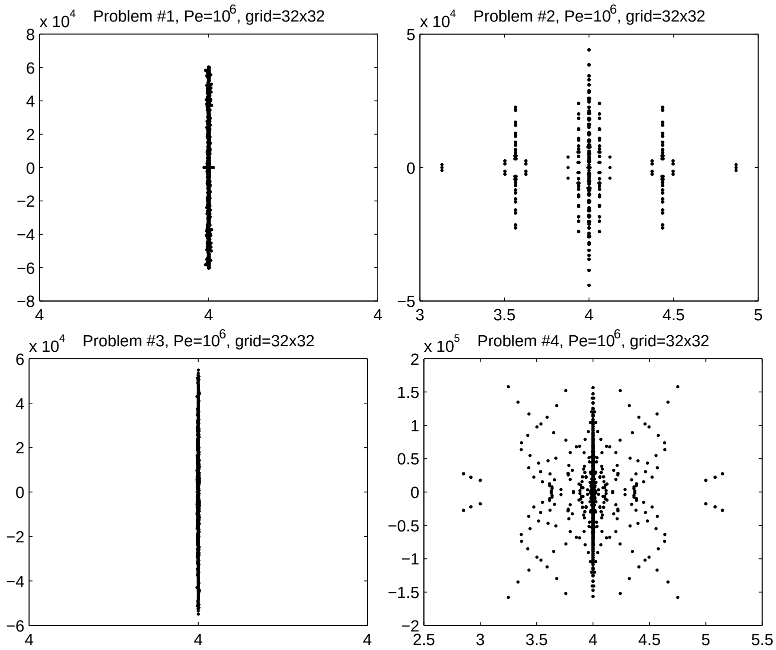
FIGURE 1. The eigenvalue distributions of the original matrix



can be obtained by using the central FD schemes [4] for first order derivatives. The upwind scheme yields M-matrix, and classical iterative methods converge in this case. We have used central difference approximation. In this case classical iterative methods for solution the resulted linear system may not converge when the Peclet number is greater than a certain constant.

To discretize the domain a mesh with meshsize h in both x and y direction was used.

When using natural ordering of unknowns, we have obtained system of linear equations with non-symmetric positive real matrix:

$$Au = f \tag{2}$$

where A is $(N-1) \times (N-1)$ matrix, u is the vector of unknown, f is the right-hand side.

In Figure 1 we depict the eigenvalue distribution of the matrix A obtained from approximation of equation (1) with various velocity coefficients (see Table 1) in order to compare it with the spectra of preconditioned matrices. We can see that the spectra of the matrices obtained from problems 1 and 3 have the same structure. All four spectrums are symmetric with respect to the point $(4, 0)$.

In this paper we present results of a preconditioned iterative solver based on BiCG for (2). First of all we compare GMRES(10) and BiCG. Further we compare the preconditioned BiCG with unpreconditioned BiCG. For completeness, we compared preconditioners proposed by us with popular SSOR precontitioner. The numerical tests were carried out on the grids 32×32, 64×64, 128×128 for all four problems (see Table 1). These test problems were borrowed from [1, 3]. The right-hand side function $f(x, y)$ was prescribed to satisfy the given exact solution $u(x, y) = e^{xy} \sin(\pi x) \sin(\pi y)$. Pe was altered between 10 and 10^6. According to the conventional classification [3] when $Pe \leq 10^3$ we get a moderately non-symmetric