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TOWARDS A LEVEL SET FRAMEWORK FOR INFARCTION MODELING: AN INVERSE PROBLEM

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Abstract. The purpose of this paper is to introduce a level set framework suitable for identifying heart infarctions. We do this by introducing a modified Monodomain model, and an output least squares formulation comparing simulation results with observation data. In this way we obtain a flexible methodology allowing us to approximately determine the characteristics of infarctions with rather complex geometrical properties. Our approach involves a CPU demanding minimization problem. This problem is solved by applying an adjoint equation enabling efficient differentiation of the involved cost-functional. Finally, our theoretical findings are illuminated by a series of numerical experiments.

Key Words. Level set, Monodomain model, heart infarctions and inverse problems.

1. Introduction

Heart conditions are among the most widespread illnesses in the Western world, and cause the deaths of millions of people every year. The main purpose of this paper is to study one of these conditions, heart infarctions; or more generally, ischemic heart illnesses. Our approach will not follow the methods of contemporary medical research, but will instead investigate the possibilities for using modern computational mathematics and high speed computers to gain insight into this growing health problem.

When a coronary artery, which supplies blood to the heart, becomes blocked, the heart muscle does not receive adequate blood and oxygen. Ultimately, this can result in the death of the heart tissue, a condition known as a heart infarction. Ischemia is a precursor of infarction, i.e. if the ischemic condition (lack of blood) persists, the outcome will be an infarction. Therefore, one might characterize an ischemia as a mild and reversible form of infarction. For the sake of simplicity, we will throughout this text use these two terms synonymously.

The first electrocardiogram (ECG) was recorded by Waller in 1887. It is difficult to exaggerate the importance of this discovery. Nowadays, millions of ECGs are recorded every day around the world. It is by far the most commonly used tool to identify ischemic heart disease. This is probably due to its simplicity, reliability, and the relatively low costs associated with buying and maintaining an electrocardiograph. However, despite its apparent success, the traditional human expert-based procedure for interpreting ECG recordings has its weaknesses; in many cases the

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procedure simply fails to detect an ischemia, see Lau et.al. [22]. Moreover, ECG recordings provide only a rather crude picture of the position and size of infarctions, cf. Birnbaum and Drew [2]. Thus, there is a need to improve this technology further.

The electrical activity in the human heart is governed by the Bidomain equations. This model was introduced by Tung [40] and has been studied by many authors; see [28, 25, 24] and references given therein. A general introduction is presented in [20].

In [38] a rather abstract and general method for including the effect of ischemic heart disease into the Bidomain equations was suggested. Furthermore, a methodology for identifying heart infarctions, in terms of an output least squares framework, was proposed and analyzed. However, among several unsolved challenges, this study did not provide any answer to the following important problem:

(1) *How can the possibly complex geometrical properties of infarctions be incorporated into such mathematical models?*

The purpose of the present paper is to shed some light onto this problem. Some initial considerations on how to handle (1) for a rather idealized model will be presented. More precisely, we propose to apply a level set methodology to include the effect of ischemic tissues into the equations. This enables us to formulate a flexible framework for identifying heart infarctions. Expressed in mathematical terms, the ischemic region is recovered by solving an inverse problem for a partial differential equation (PDE). That is, boundary measurements of the solution of the PDE is applied to identify an unknown coefficient in the equation. This unknown coefficient, as well as the involved conductivities, are parameterized in terms of a level set technique.

The Bidomain model consists of a coupled system of PDEs. Their efficient numerical solution is a very difficult scientific issue, typically involving extremely CPU demanding algorithms - see [37]. As mentioned above, the main purpose of the present paper is to propose a method for incorporating the possibly complex geometrical structures of infarctions into models of the electrical activity in the heart. For the sake of simplicity, we will therefore focus on the less physically accurate Monodomain model[1]. The Monodomain model consists of a scalar parabolic PDE. It can therefore be solved efficiently by standard numerical methods for parabolic problems. Hence, this equation is well-suited for a conceptually study of the problem posed in (1).

Several scientists have analyzed inverse problems that arise in connection with ECG recordings. In particular, the problem of computing the epicardial potential, i.e. the electrical potential at the surface of the heart, from body surface measurements has received a lot of attention. (See, e.g. Franzone, Taccardi and Viganotti [10], MacLeod and Brooks [27], Dössel [7] and Greensite and Huiskamp [11], to mention a few.) Roughly speaking, the goal of such studies is to compute ECG recordings for the surface of the heart, and thereby obtain a deeper understanding of this organ. This task may be formulated in terms of a single linear elliptic partial

[1]The Monodomain model can be derived from the Bidomain equations by assuming that the involved extra and intra -cellular conductivities are proportional quantities, see [20, 38] for further details. We hope to treat the Bidomain case in our future work.