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CORRIGENDUM: A POSTERIORI ERROR ESTIMATION FOR NON-CONFORMING QUADRILATERAL FINITE ELEMENTS

MARK AINSWORTH

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Abstract. An invalid assumption on the equivalence of alternative sets of degrees of freedom for the rotated $\mathbb{Q}_1$ element led to an incompatibility between the analysis and the application in our earlier article [1]. We outline minor modifications needed to restore the validity of all results and conclusions in the previous article.

Key Words.

1. Introduction

In an earlier article [1], we presented an a posteriori error bound for finite element approximation using the non-conforming rotated $\mathbb{Q}_1$ element. This element may be viewed as a triple (S, P, Σ), where S is the reference square, P is the approximation space $\{1, \hat{x}, \hat{y}, \hat{x}^2 - \hat{y}^2\}$ and Σ is a unisolvent set of degrees of freedom that may be chosen in two distinct ways: either (**C**) function evaluation at the midpoints of the sides of S, or (**C** ′) line integrals over the sides of S. However, as pointed out in [2], the actual finite element approximation is *not* the same for both sets of degrees of freedom[1]! This means that the analysis in [1] is not applicable in case (**C**) but, after some minor modifications which we outline below, is applicable in case (**C** ′).

2. Modifications

The spaces $X_{\mathcal{P}}$ and $X_{\mathcal{P},E}$ appearing in [1, p. 4] should be changed to read

$$X_{\mathcal{P}} = \left\{ v : \Omega \to \mathbb{R} : v_{|K} \circ \boldsymbol{F}_K \in P \quad \forall K \in \mathcal{P}, \quad \int_\gamma [v] \,\mathrm{d}s = 0 \quad \forall \gamma \in \partial\mathcal{P} \backslash \partial\Omega \right\}$$

where $[v]$ denotes the jump across an interface, with the subspace $X_{\mathcal{P},E}$ defined by

$$X_{\mathcal{P},E} = \left\{ v \in X_{\mathcal{P}} : \int_\gamma v \,\mathrm{d}s = 0 \text{ for } \gamma \subset \Gamma_D \right\}.$$

With this modification, the range of the operator $\Pi_{\mathcal{P}}$ defined in [1, eq.(6)] is the space $X_{\mathcal{P},E}$. This would not be the case with the degrees of freedom chosen as in (**C**) and is the reason why the previous analysis is not applicable to (**C**). The remaining analysis in [1] is then correct as written as far as [1, eq.(45)] which should be replaced by

$$u^*(\boldsymbol{m}_\gamma) = \begin{cases} 0, & \text{if } \boldsymbol{m}_\gamma \in \Gamma_D \\ h_\gamma^{-1} \int_\gamma u_{\mathcal{P}} \,\mathrm{d}s, & \text{otherwise.} \end{cases}$$

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[1]The author is indebted to Professor C. Carstensen (Berlin) for drawing his attention to this fact.

The choice of degrees of freedom according to ($\mathbf{C}\,'$) means that the values of the function u^* at the midpoints are well-defined by this formula. This modified definition means the estimate [1, eq.(51)] is now proved as follows. Firstly, thanks to the modified definition of $X_{\mathcal{P},E}$,

$$|u_{\mathcal{P}}(\boldsymbol{x}_n)_{|K'} - u_{\mathcal{P}}(\boldsymbol{x}_n)_{|K}| = |[u_{\mathcal{P}}(\boldsymbol{x}_n)] = \left|[u_{\mathcal{P}}(\boldsymbol{x}_n)] - h_\gamma^{-1}\int_\gamma [u_{\mathcal{P}}]\,\mathrm{d}s\right|$$

and then, using the following identity

$$w(\boldsymbol{x}_n) - h_\gamma^{-1}\int_\gamma w\,\mathrm{d}s = h_\gamma^{-1}\int_\gamma \mathrm{d}s\int_s^{\boldsymbol{x}_n}\mathrm{d}\tau\,\frac{\partial w}{\partial \tau},$$

along with a Cauchy-Schwarz inequality, we derive the estimate

$$\left|[u_{\mathcal{P}}(\boldsymbol{x}_n)] - h_\gamma^{-1}\int_\gamma [u_{\mathcal{P}}]\,\mathrm{d}s\right| \le C h_\gamma^{1/2}\|J_\gamma^\tau\|_\gamma$$

which gives the same estimate as [1, eq.(51)]. A similar modification is needed to obtain estimate [1, eq.(52)].

Finally, the definition of the function g_K on an interior edge γ, given on Page 6, should be should be modified to read

$$g_K = \begin{cases} \dfrac{1}{2|\gamma|}\displaystyle\int_\gamma \boldsymbol{n}_K\cdot\left(a_K\,\mathbf{grad}_{\mathcal{P}}\,u_{\mathcal{P}}|_K + a_{K'}\,\mathbf{grad}_{\mathcal{P}}\,u_{\mathcal{P}}|_{K'}\right)\mathrm{d}s & \text{on } \gamma = \partial K\cap\partial K' \\ \dfrac{1}{|\gamma|}\displaystyle\int_\gamma \boldsymbol{n}_K\cdot a_K\,\mathbf{grad}_{\mathcal{P}}\,u_{\mathcal{P}}|_K\,\mathrm{d}s & \text{on } \gamma\subset\Gamma_D \end{cases}$$

which then leads to equation (19) holding with the jump residual J^ν defined on the boundary of element K by

$$-\frac{1}{2}J^\nu_{|\gamma} = g_K - \boldsymbol{n}_K\cdot a_K\,\mathbf{grad}_{\mathcal{P}}\,u_{\mathcal{P}}|_K\,.$$

The remainder of the analysis in [1] then applies without further modification.

In summary, with these modifications, all analysis and conclusions given in the previous work [1] are correct.

References

[1] Mark Ainsworth. A posteriori error estimation for non-conforming quadrilateral finite elements. *Int. J. Numer. Anal. Model.*, 2(1):1–18, 2005.

[2] Carsten Carstensen. Personal communication. 2006.

Mathematics Department, Strathclyde University, 26 Richmond Street, Glasgow G1 1XH, Scotland.

E-mail: M.Ainsworth@strath.ac.uk

URL: http://www.maths.strath.ac.uk/∼aas98107