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ALTERNATING SCHEMES OF PARALLEL COMPUTATION FOR THE DIFFUSION PROBLEMS

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Abstract. In this paper, a set of new alternating segment explicit-implicit (NASEI) schemes is derived based on an one-dimensional diffusion problem. The schemes are capable of parallel computation; third-order accurate in space; and stable under a reasonable mesh condition. The numerical examples show that the NASEI schemes are more accurate than either the old ASEI or the ASCN schemes.

Key Words. New alternating segment explicit-implicit (NASEI) schemes, finite difference method; diffusion problems, parallel computation.

1. Introduction

The goal of this paper is to study appropriate finite difference schemes suitable for parallel computation. Two major types of schemes which are capable of parallel computation are: the alternating schemes ([1]-[3]), and the domain decomposition schemes ([4]-[6]). Our interest is on the alternating schemes which the NASEI schemes belong.

Before getting into the detail construction of the NASEI schemes, we like to briefly mention three closely related existing alternating schemes. They are: the Alternating Group Explicit (AGE) schemes ([1]); the Alternating Segment Explicit-Implicit (ASEI) schemes ([2]); and the Alternating Segment Crank-Nicolson (ASCN) schemes ([3]). All these three schemes are capable of parallel computation, however, their truncation errors are only second order or lower. Thus, we propose to derive the NASEI schemes, a new set of alternating schemes, which are capable of parallel computation; stable under a reasonable condition; and have truncation errors of third order.

The NASEI schemes are derived based on the following diffusion problem with periodic solution:

$$Lu \equiv \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0,\ x \in \Re,\ t \in [0,T], \tag{1.1}$$

$$u(x,t) = u(x+H,t),\ x \in \Re,\ t \in [0,T], \tag{1.2}$$

$$u(x,0) = u_0(x),\ x \in \Re. \tag{1.3}$$

Here, H represents the length of one period.

The outline of the paper is as following. Six basic schemes of (1.1) are introduced in Section 2. The NASEI schemes are derived; their stability result is proved; and

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their truncation errors are obtained, all in Section 3. The numerical examples are presented in Section 4. Finally, a short conclusion remark is given in Section 5.

2. The six basic schemes

2.1. Preliminaries. The idea of using basic schemes to derive alternating schemes was first used by us in ([3]), where six basic schemes were introduced to derive a set of alternating schemes for the Dispersive equation. In this paper, we generalize this idea to the diffusion equation (1.1).

Throughout the rest of the paper, $\triangle x$ and $\triangle t$ are used to represent the spacial mesh size and time increment respectively; r represents $\frac{\triangle t}{\triangle x^2}$; and h represents the pair $(\triangle x,\, \triangle t)$. U_j^n is used to represent the approximate value of $u(x_j, t^n)$ which is shortened to u_j^n. Here $u(x,\, t)$ represents the exact solution. We assume that there exists a positive integer J, such that $J\,\triangle x = H$.

2.2. The six basic schemes. The first two basic schemes are the following explicit and implicit schemes:

$$U_j^{n+1} + \frac{r}{12}\,U_{j-2}^n - \frac{4r}{3}\,U_{j-1}^n - (1 - \frac{5r}{2})\,U_j^n - \frac{4r}{3}\,U_{j+1}^n + \frac{r}{12}\,U_{j+2}^n = 0, \tag{2.1}$$

$$\frac{r}{12}\,U_{j-2}^{n+1} - \frac{4r}{3}\,U_{j-1}^{n+1} + (1 + \frac{5r}{2})\,U_j^{n+1} - \frac{4r}{3}\,U_{j+1}^{n+1} + \frac{r}{12}\,U_{j+2}^{n+1} - U_j^n = 0. \tag{2.2}$$

The other four are four asymmetric schemes given below, their rules are displayed at the end of the paper.

$$\begin{aligned} & (1 + \frac{7r}{12})\,U_j^{n+1} - \frac{2r}{3}\,U_{j+1}^{n+1} + \frac{r}{12}\,U_{j+2}^{n+1} \\ = \; & -\frac{r}{12}\,U_{j-2}^n + \frac{4r}{3}\,U_{j-1}^n + (1 - \frac{23r}{12})\,U_j^n + \frac{2r}{3}\,U_{j+1}^n, \end{aligned} \tag{2.3}$$

$$\begin{aligned} & -\frac{2r}{3}\,U_{j-1}^{n+1} + (1 + \frac{23r}{12})\,U_j^{n+1} - \frac{4r}{3}\,U_{j+1}^{n+1} + \frac{r}{12}\,U_{j+2}^{n+1} \\ = \; & -\frac{r}{12}\,U_{j-2}^n + \frac{2r}{3}\,U_{j-1}^n + (1 - \frac{7r}{12})\,U_j^n, \end{aligned} \tag{2.4}$$

$$\begin{aligned} & \frac{r}{12}\,U_{j-2}^{n+1} - \frac{4r}{3}\,U_{j-1}^{n+1} + (1 + \frac{23r}{12})\,U_j^{n+1} - \frac{2r}{3}\,U_{j+1}^{n+1} \\ = \; & (1 - \frac{7r}{12})\,U_j^n + \frac{2r}{3}\,U_{j+1}^n - \frac{r}{12}\,U_{j+2}^n, \end{aligned} \tag{2.5}$$

$$\begin{aligned} & \frac{r}{12}\,U_{j-2}^{n+1} - \frac{2r}{3}\,U_{j-1}^{n+1} + (1 + \frac{7r}{12})\,U_j^{n+1} \\ = \; & \frac{2r}{3}\,U_{j-1}^n + (1 - \frac{23r}{12})\,U_j^n + \frac{4r}{3}\,U_{j+1}^n - \frac{r}{12}\,U_{j+2}^n. \end{aligned} \tag{2.6}$$

If $L_h^{(2.1)}$, $L_h^{(2.2)}$, $L_h^{(2.3)}$, $L_h^{(2.4)}$, $L_h^{(2.5)}$, $L_h^{(2.6)}$ are used to represent the analogous discretizied operators of L based on schemes (2.1)-(2.6), then their truncation errors