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A MIXED-INTEGER PROGRAMMING APPROACH TO NETWORKED CONTROL SYSTEMS

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Abstract. This paper studies the problem of controller design for networked control systems regulated by a network data transmission protocol proposed in [50]. In this framework, the plant is first formulated as a mixed logical dynamical (MLD) system, then model predictive control (MPC) based on the mixed-integer programming is adopted to design a controller to guarantee certain control performance. It is shown that the solvability of the finite-horizon MPC is not equivalent to that of the infinite-horizon MPC, which is normally true for most existing MPC methods. The non-convexity feature of this type of networked control systems rules out explicit piecewise affine controllers that are designable for linear convex control systems. Notwithstanding these difficulties, controller design is still feasible due to the special nature of the data transmission strategy, i.e., only a small number of logic values are involved. Furthermore, control of higher-order systems and tracking of more complicated signals can be readily dealt with using this new approach. Two examples are presented to illustrate the strength of the proposed approach.

Key Words. model predictive control, networked control systems, non-convexity, mixed-integer programming, mixed logical dynamical systems, hybrid systems.

1. INTRODUCTION

The fast development of secure high-speed communication networks ([40, 29]) renders control over networks possible. The insertion of a communication channel into a control loop brings in many advantages, including wire reduction, low cost and easy installation and maintenance, etc.. Thanks to these merits, networked control systems have been built successfully in various fields like automotive control ([18, 28]), aircrafts manufacturing ([33, 36]), and robotic control ([20, 35]). Unfortunately, since most types of signals, like the encoded system output, controller output and other information, are transmitted through communication channels shared by a lot of users, traffic congestion always occurs which of course degrades control performance. Traffic congestion usually manifests itself in the form of time

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delays, packet loss, and other undesirable effects on the control systems. How to tackle these adverse effects has become a major subject of research in the control community and such closely related fields as communication and computing. A variety of network protocols and lots of control techniques have already been proposed and analyzed to tackle this issue. To classify very crudely, these considerations can be categorized into three classes.

The first category contains simplest approximations where networked control systems are modeled as feedback control systems having bounded deterministic time delays. For instance, a so-called try-once-discard (TOD) protocol is recently proposed and studied extensively by Walsh *et al.* ([41, 42, 43, 44, 45]). In this protocol, it is assumed there is no network from controller to actuator, while an upper bound of sensor-to-controller time delays induced by the network is obtained based on the perturbation theory, for which the resulting closed-loop system is exponentially stable. This protocol is further developed in [24], [25] to establish a class of Lyapunov UGES (Uniformly Globally Exponentially Stable) protocols in the L^p space framework. In reference [48], by assuming bounded time delays and packet dropouts, the authors investigate a robust H_∞ control problem for networked control systems. However, all the above papers did not study the issue of time delays or packet loss from controllers to actuators. A possible difficulty in dealing with controller-to-actuator delays and packet loss may be: A pre-defined controller is usually unable to predict and then compensate for time delays or packet loss from controllers to actuators. Based on the above discussion, techniques in this class usually are quite conservative because of their inherent limitations in system modeling, as has been widely acknowledged.

The foregoing conservativeness has triggered the development of the second category of methods, where network-induced time delays and packet loss are modeled as random processes, typically Markov chains. Via this modeling, specific properties of these stochastic processes can be deployed to facilitate the design of controllers guaranteeing desirable control performance. For instance, in [18], by assuming time delays as Markov chains, a jumped linear system is constructed via state augmentation. Moreover, several necessary and sufficient conditions for zero-state mean-square exponential stability have been established for this type of networked control systems. A similar approach is adopted in [49] where sensor-to-controller and controller-to-actuator time delays are assumed to behave according to two Markov chains respectively. LMI techniques are then employed to address the stabilization problem. In [26], both sensor-to-controller and controller-to-actuator time delays are modeled as independent white-noise with zero mean and unit variance; consequently, a (sub)optimal stochastic control problem is formulated and studied. Two Matlab toolboxes, Jitterbug and TrueTime, are proposed recently in [8] based on the assumption that a networked control system can be approximated by a sampled-data control system with signal quantization and time delays involved. These two toolboxes can be readily adopted to determine how sensitive a control