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ALGORITHMS FOR COUPLED MECHANICAL DEFORMATIONS AND FLUID FLOW IN A POROUS MEDIUM WITH DIFFERENT TIME SCALES

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Abstract. In this paper, we solve a problem describing the mechanical deformations of a porous medium in the presence of a monophasic linear flow or a two phase nonlinear flow with the purpose of modelizing subsidence of hydrocarbon reservoirs. An essential characteristics of this problem is that the mechanical deformation and the flow have different time scales. In petroleum industry, one uses different very efficient simulators for the flow problem and the mechanical deformations, which enables to handle complex models. Therefore it is necessary to be able to combine as efficiently as possible the exploitation of these simulators. We propose two alternative splitting approaches. The first one is the staggered algorithm used by engineers, which amounts to a Gauss-Seidel method in the one phase linear case. The second approach is based upon the preconditioned conjugate gradient method. We use a numerical multi-scale method in both of these algorithms. We compare these two approaches and we show that the preconditioned conjugate gradient algorithm is faster and more robust than the staggered algorithm. Applying the preconditioned conjugate gradient algorithm therefore seems to compensate for the fact that the inf-sup condition for the mixed discretization method is not satisfied when combining the simulators for the mechanical deformations and for the flow computations.

Key Words. Porous media, Darcy flow, Mechanical deformations, Gauss Seidel method, Nonlinear conjugate gradient method, Inf-Sup condition, Mixed formulation, Multiscale algorithm.

1. Introduction

The production of oil and gas in soft highly compacting reservoirs induces an important reduction of the pore volume, which increases the oil productivity. This compaction leads to undesirable effects such as surface subsidence or damage of well equipments. A well- known example of subsidence is the Ekofisk field in the North Sea in Norway, where a sea floor subsidence rate of 42 $cm/year$ has been reached at the end of 1993 (see [11]). The cases of the Valhall field in Norway (see [9]) and the Bachaquero (see [8]) and Tia Juana (see [7]) fields in Venezuela also illustrate the importance of the subsidence phenomenon in oil production. The purpose of this paper is to simulate the mechanical deformations of the porous media in the presence of a Darcy flow in two space dimensions, taking into account that they

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have different time scales.

The behavior of the skeleton is described through a linear elastic equation, whereas we consider two alternative flow models, a monophasic linear case and a two phase nonlinear one. The coupling between these equations is given by the Biot's law which connects the variation of the porosity of the ground to the variation of the mechanical deformations. The main physical unknowns for the linear monophasic flow are the pressure and the porosity; as for the two phase nonlinear flow, the unknown functions are the pressure, saturation and porosity, and the geomechanical problem is written in terms of the displacement. A possible way is to write a complete simulator involving all the flow and mechanical unknowns (see [2]), but this seems to be too expensive. In practice the variation of the displacement is much smaller than the variation of the flow unknowns which means that we are dealing with a multi-scale phenomenon. In the petroleum industry, there already exist different simulators which permit to separately solve the flow problem and the geomechanical one, and since those simulators are very efficient and able to handle complex models, it seems natural to combine them. Typically the flow simulators are based upon a finite volume method whereas one applies a standard finite element method for the discretization of the geomechanical problem. In practice, one has to deal with the fact that geomechanical simulators are more expensive than flow simulators; further one should keep in mind the fact that the inf-sup condition is not satisfied by the mixed discretization method imposed by using the separate simulators. The purpose of this work is to compare two splitting approaches in the case of a two dimensional prototype, extending a previous study performed in a one-dimensional case (see [4]):

- the first one, which has been introduced by engineers, is based upon the computation of the flow unknowns and the displacement by means of a fixed point method; more precisely one makes use of an iterative procedure and we show that in the monophasic linear case this amounts to a Gauss-Seidel type iteration method;
- the new splitting approach which we propose is based upon a preconditioned conjugate gradient method.

In fact, we use a multi-scale coupling algorithm where we study the impact of ratio of the time steps of the mechanical and fluid flow computations on the precision of the numerical results.

The new algorithm turns out to be more robust than the first one; in particular it works fine even though the Inf-Sup condition is not satisfied in the mixed discretization method and it is faster and less expensive. Note that the precision of the computations is satisfactory as soon as the ratio of the two time steps is not too large. These approaches are applied in both the cases where the mechanical deformations of the ground are coupled to a mononphasic linear flow and to a two phase nonlinear flow.

We present the two problems in Section 2. We will refer to the model with the monophasic linear flow as to the linear model and we will call nonlinear model the system involving the nonlinear two phase flow. In Section 3 we discretize the two models and present the complete discretized problems. In Section 4 we show that the Inf-Sup condition is not satisfied because of the mixed discretization which is imposed by using separate simulators. We introduce in Section 5 the first splitting approach which we refer to as the multi-scale staggered algorithm and we show that