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USING REDUCED MESHES FOR SIMULATIONS OF THE LOCALIZATION OF SMALL ELECTROMAGNETIC INHOMOGENEITIES IN A 3D BOUNDED DOMAIN

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Abstract. We are concerned in this work with simulations of the localization of a finite number of small electromagnetic inhomogeneities contained in a three-dimensional bounded domain. Typically, the underlying inverse problem considers the time-harmonic Maxwell equations formulated in electric field in this domain and attempts, from a finite number of boundary measurements, to localize these inhomogeneities. Our simulations are based on an approach that combines an asymptotic formula for perturbations in the electromagnetic fields, a suited inversion process, and finite element meshes derived from a non-standard discretization process of the domain. As opposed to a recent work, where the usual discretization process of the domain was employed in the computations, here we localize inhomogeneities that are one order of magnitude smaller.

Key words. inverse problems, Maxwell equations, electric fields, inhomogeneities, Current Projection method, MUSIC method, FFT, edge elements, numerical measurements, composite numerical integrations.

1. Introduction

This work falls directly in the field of Electrical Impedance Tomography. We seek to recover unknown inhomogeneities contained in a bounded domain from a finite number of measurements evaluated on its boundary. From a practical point of view, such measurements are experimental (or physical) whereas from a simulation point of view, they are numerically evaluated. Usually in this simulation context, we solve the underlying inverse problem with the help of a localization procedure that considers, as data, numerical boundary measurements. Typically, each one of these measurements results from a numerical computation of the physical field present in the domain, due to a current applied on its boundary.

In simulations of the localization of small electromagnetic inhomogeneities contained in a three-dimensional bounded domain, we must for instance compute by a finite element method the electric (or magnetic) field, induced by each prescribed boundary current, in order to evaluate numerically the corresponding boundary measurement of "voltage" type. When the required finite element method is based on the usual triangulation process of the domain, we are concerned, for each prescribed boundary current, with a discrete formulation in electric (or magnetic) field which is numerically expensive to solve. In fact, the usual triangulation process generates a "full" conforming mesh of the domain that implicitly takes into account the discretization of each inhomogeneity and leads to a very large number of degrees of freedom caused by the smallness of the inhomogeneities — especially as this is a three-dimensional domain and as mixed finite elements are considered. The discrete system deriving from the afore-mentioned formulation then has a very

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large number of unknowns and even by solving this system with preconditioning techniques, we observe, as in [6], that the CPU time needed to evaluate numerically each boundary measurement remains important. In the presence of a large number of small inhomogeneities, the number of degrees of freedom associated with the discrete formulation is excessive and can forbid numerical simulations due now to the exorbitant requirements in memory storage. Considering then a full conforming mesh of the domain when it contains multiple small inhomogeneities leads to some drastic drawbacks regarding the numerical localization as far as memory storage and CPU time are concerned.

Here we are interested in simulations of the localization of small electromagnetic inhomogeneities in a three-dimensional bounded domain, based on finite element meshes that derive from a non-standard discretization process of the domain. This process is aimed at overcoming the drawbacks inherent in full meshes.

As opposed to [6], where full meshes were considered for the localization of inhomogeneities and where we were limited in simulations by the smallness of the inhomogeneities, we expect here to be able to perform localization of much smaller inhomogeneities.

Our approach will also be based on the framework recently proposed by H. Ammari, M.S. Vogelius & D. Volkov [4]. Typically, this framework considers the time-harmonic Maxwell equations in a three-dimensional bounded domain Ω containing a finite number m of unknown inhomogeneities of small volume, and proposes to localize these inhomogeneities from an asymptotic expansion of the perturbation in the (tangential) boundary magnetic field. In the presence of well-separated inhomogeneities, and also distant from $\partial\Omega$, the boundary of Ω, the asymptotic expansion states that, for any $z \in \partial\Omega$,

(1)

$$\begin{aligned}(H_\alpha - H_0)(z) \times \nu(z) \,-\, 2\int_{\partial\Omega} \mathrm{curl}_z(\Phi^k(x,z)(H_\alpha - H_0)(x) \times \nu(x)) \times \nu(z)\, d\sigma_x \\ = \, 2\alpha^3\omega^2 \sum_{j=1}^{m} \frac{\mu_0}{\mu_j}(\mu_0 - \mu_j) G(z_j, z) \times \nu(z) M^j(\frac{\mu_0}{\mu_j}) H_0(z_j) \\ + \, 2\alpha^3 \sum_{j=1}^{m} (\frac{1}{\varepsilon_j} - \frac{1}{\varepsilon_0})((\mathrm{curl}_x\, G)(z_j, z))^T \times \nu(z) M^j(\frac{\varepsilon_0}{\varepsilon_j})(\mathrm{curl}_x\, H_0)(z_j) + \mathrm{O}(\alpha^4)\,.\end{aligned}$$

In (1), α is the common order of magnitude of the diameters of the inhomogeneities, and the points z_j, $1 \le j \le m$, represent the 'centers' of the inhomogeneities. The magnetic field is denoted by H_α in the presence of the inhomogeneities and by H_0 in the absence of inhomogeneities. The outward unit normal to Ω is represented by ν, and ω is a given frequency. The (constant) background magnetic permeability and complex permittivity are μ_0 and ε_0 respectively. Also, μ_j and ε_j are the (constant) magnetic permeability and the complex permittivity of the jth inhomogeneity, $k^2 = \omega^2\varepsilon_0\mu_0$, Φ^k is the "free space" Green's function for the Helmholtz operator $\Delta + k^2$. The operators applied to the matrix valued function G act column-by-column, and $G(x,z)$ is the "free space" Green's function for the "background" magnetic problem: $\mathrm{curl}_x\,(\frac{1}{\varepsilon_0}\,\mathrm{curl}_x G(x,z)) - \omega^2\mu_0 G(x,z) = -\delta_z \mathrm{I}_3$, with I_3 the 3×3 identity matrix, δ_z the Dirac delta at z. Also in (1), the superscript "T" denotes the transpose, $M^j(\frac{\mu_0}{\mu_j})$ and $M^j(\frac{\varepsilon_0}{\varepsilon_j})$ are the polarization tensors associated with the jth inhomogeneity (symmetric 3×3 matrices). Finally, the notation $\mathrm{O}(\alpha^4)$