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MULTISCALE ASYMPTOTIC METHOD FOR HEAT TRANSFER EQUATIONS IN LATTICE-TYPE STRUCTURES

FANG-MAN ZHAI AND LI-QUN CAO

Abstract. In this paper, we discuss the initial-boundary value problem for the heat transfer equation in lattice-type structures that arises from the aerospace industry and the structural engineering. The main results obtained in this paper are the convergence theorems by using the homogenization method and the multiscale asymptotic method (see Theorems 2.1 and 2.2). Some numerical examples are given for three types of lattice structures. These numerical results suggest that the first-order multiscale method should be a better choice compared with the homogenization method and the second-order multiscale method for solving the heat transfer equations in lattice-type structures.

Key Words. homogenization, multiscale asymptotic expansion, parabolic equation, lattice-type structure, multiscale finite element method.

1. Introduction

In this paper, we consider the initial-boundary value problem for second order parabolic equations with rapidly oscillating coefficients as follows

$$
(1)\qquad \begin{cases} \dfrac{\partial u^{\varepsilon\delta}(x,t)}{\partial t} - \dfrac{\partial}{\partial x_i}\Big(a_{ij}(\frac{x}{\varepsilon},t)\dfrac{\partial u^{\varepsilon\delta}(x,t)}{\partial x_j}\Big) = f(x,t),\ (x,t)\in\Omega_{\varepsilon\delta}\times(0,T) \\ u^{\varepsilon\delta}(x,t) = g(x,t), \quad (x,t)\in\partial\Omega\times(0,T) \\ \nu_i a_{ij}(\frac{x}{\varepsilon},t)\dfrac{\partial u^{\varepsilon\delta}}{\partial x_j} = 0, \quad (x,t)\in\partial T_{\varepsilon\delta}\times(0,T) \\ u^{\varepsilon\delta}(x,0) = \bar{u}_0(x), \end{cases}
$$

where $f(x,t)$, $g(x,t)$, $\bar{u}_0(x)$ are some known functions. We follow Cioranescu's notation (see [6], p.74) by denoting $\Omega_{\varepsilon\delta} = \Omega\setminus\overline{T}_{\varepsilon\delta}$ the perforated domain, where Ω is a bounded domain of $R^n, n\geq 2$, $T_{\varepsilon\delta} = \tau(\varepsilon T_\delta)$ is the set of all translated images of $\varepsilon\overline{T_\delta}$ of the form $\varepsilon(z+\overline{T_\delta})$, $z=(z_1,\cdots,z_n)\in Z^n$, $T_\delta = Y\setminus Y_\delta$, $Y=(0,1)^n$. Here Y_δ is a lattice structure as shown in Figs. 1-3. The boundaries of Ω and $T_{\varepsilon\delta}$ are respectively $\partial\Omega$ and $\partial T_{\varepsilon\delta}$ and $\vec{\nu}=(\nu_1,\cdots,\nu_n)$ is the unit outer normal to $\partial T_{\varepsilon\delta}$.

In order to apply the extension theorem (see, e.g. Theorem 2.10 of [6], p. 28), we make the following assumption

($\mathbf{H_1}$) The holes do not intersect the boundary $\partial\Omega$.

This assumption restricts the geometry of the open set Ω. For example, Ω can be a finite union of the periodic cells.

Set $\xi=\varepsilon^{-1}x$, and suppose that

($\mathbf{A_1}$) the coefficients $a_{ij}(\xi,t)$ are 1-periodic in ξ;

($\mathbf{A_2}$) $\gamma_0|\eta|^2\leq a_{ij}(\xi,t)\eta_i\eta_j\leq\gamma_1|\eta|^2, \quad \forall(\eta_1,\cdots,\eta_n)\in R^n$,
γ_0, γ_1 are positive constants independent of ε;

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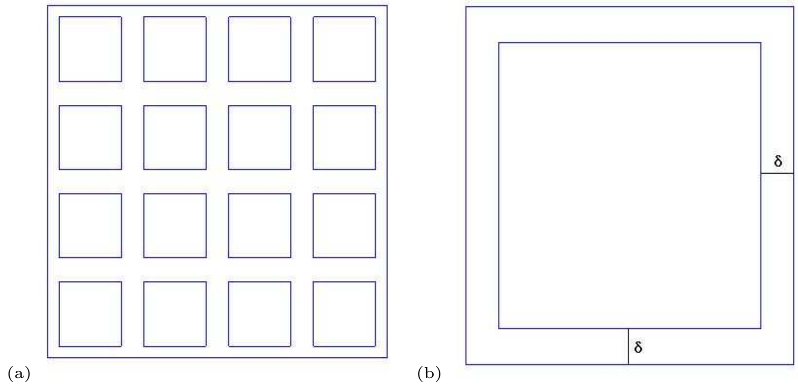

FIGURE 1. (a) lattice-type structure: Type I; (b) the unit cell Y_δ.

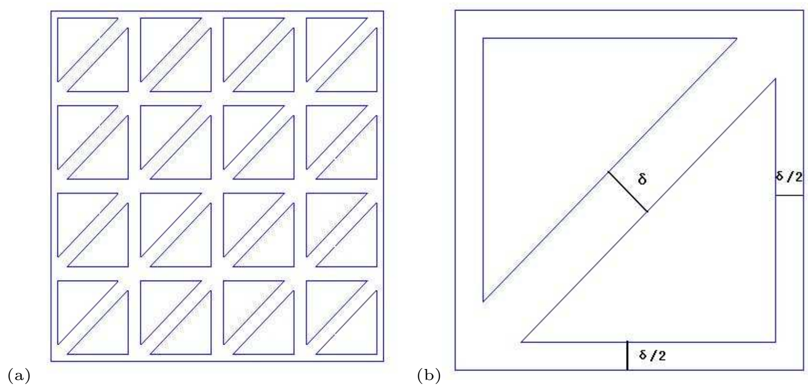

FIGURE 2. (a) lattice-type structure: Type II; (b) the unit cell Y_δ.

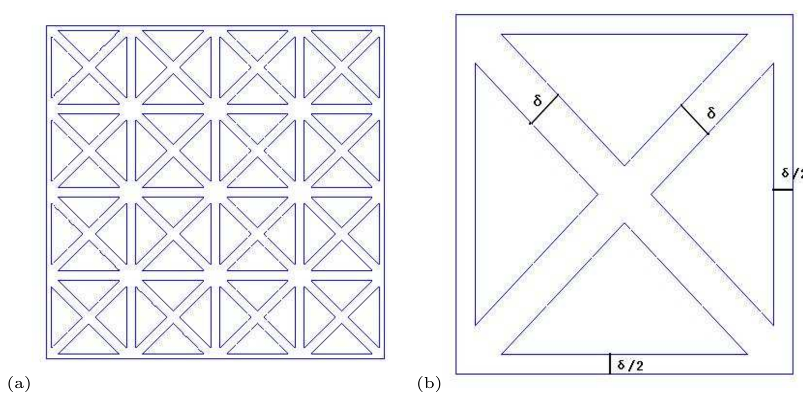

FIGURE 3. (a) lattice-type structure: Type III; (b) the unit cell Y_δ.

($\mathbf{A_3}$) $f \in L^2(0,T;L^2(\Omega)), g \in L^\infty(0,T;H^{1/2}(\partial\Omega)), \partial_t g \in L^2(0,T;H^{1/2}(\partial\Omega))$, $\bar{u}_0 \in H^1(\Omega), g(x,0) = \bar{u}_0(x)$.

Lattice-type structures are characterized by two properties: periodicity and small thickness of the material. Such structures have a wide range of applications in