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DYNAMICAL SIMULATION OF RED BLOOD CELL RHEOLOGY IN MICROVESSELS

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This paper is dedicated to Professor Roland Glowinski for his 70th birthday

Abstract. A spring model is applied to simulate the skeleton structure of the red blood cell (RBC) membrane and to study the red blood cell (RBC) rheology in microvessels. The biconcave RBC shape in static plasma and tank-treading behavior of single cell in shear flows have been successfully captured in this model. The behavior of the RBC in a Poiseuille flow and the lateral migration of the cells in a shear flow have been investigated. It is found that the RBCs exhibit parachute shape in a Poiseuille flow with the curvature closely related to the deformability of the cell membrane and the hematocrit (Hct) of the blood. With this spring model, RBCs can recover their initial shapes associated with the minimal elastic energy when the flow stops. The simulation results also show that the RBCs migrate to the center of the domain in the radial direction in a shear flow, which clearly indicates the Fahraeus-Lindqvist effect in microvessels. The rate of migration toward the center depends on the shape of the RBC; the bioconcave shape enhences this migration.

Key Words. Computational Biomechanics, Microcirculation, Rheology, Red blood cells, Elastic membrane model, Immersed boundary method.

1. Introduction

The microcirculation, which is comprised of the microvessels of diameter smaller than $100\mu m$, is essential to the human body. It is where exchange of mass and energy takes place. At the microcirculatory level, the particulate nature of the blood becomes significant. The rheological property of the red blood cells (RBCs) is a key factor of the blood flow characteristics in microvessels because of their large volume fraction (40-45%), so called hematocrit (Hct), in the whole blood. Under normal conditions, RBCs are biconcave-shaped discs of about $8\mu m$ in diameter. The cell membrane is highly deformable so that RBC can change its shape when an external force is acting on it and returns to the biconcave resting shape after the removal of the forces [13]. In microvessels having internal diameter close to the cell size,

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the RBCs exhibit well known parachute shapes under flow [31]. In a bigger microvessel, RBCs tend to move across the streamlines of the flow, so called lateral migration, to the center of the vessel so that there is a cell-free layer near the vessel wall. The non-uniform distribution of hematocrit within the cross-section of the vessel is the physical reason of Fahraeus-Lindqvist effect [12] which is characterized by a decrease in the apparent blood viscosity in such microvessels.

As in [5], *in silico* mathematical modeling is an attractive alternative since it is difficult to deal with *in vivo* and *in vitro* experiments on studying microcirculation and RBC rheology due to the size limitation. Nowadays, numerical study of RBC rheology has attracted growing interest (see, e.g., [28]). For example, in [29] the parachute shape of RBCs in capillaries was investigated with different Hct and the apparent blood viscosity in capillaries was also studied by using the boundary-integral method with both Mooney-Rivlin and Skalak models plus bending resistance for the RBC membrane. In [11], an immersed boundary method was used to simulate 3D capsule and RBCs in shear flow with both neo-Hookean and Skalak models for membrane deformation. It was found that the bending resistance must be included in order to simulate complex shape of RBCs when they deform in shear flow. In [2], an immersed boundary method and a neo-Hookean model with and without bending resistance were used to simulate the interaction of two deformable cells in a shear flow in two dimensions. It was found that aggregates made of deformable cells are easily breakable by a shear flow, while those made of less deformable cells are not. In [20, 22], an immersed finite element method was presented for the simulation of RBCs in three dimensions while the RBC membrane employing a Mooney-Rivlin model. The microscopic mechanism of RBC aggregation has been linked to the macroscopic blood viscosity via direct numerical simulation and the relation between the effective viscosity of blood flow and the diameters of capillaries has been obtained. In [33], a semi-implicit particle method combined with a spring model was used to simulate a single file of RBCs between two parallel plates for various Hct in two dimensions. The parachute shape of RBCs in capillaries and flow resistance were investigated with different Hct. In [9], a discrete model for the RBC membrane has been constructed by taking into account the volume constraint of the RBC, the local area constraint on each triangle element from the mesh for the RBC membrane, the total area constraint of the RBC surface, the stretching force between nodes on each edge of the surface triangle element, and the preferred angle between triangle elements sharing a common edge (the bending resistance). These constraints give different forces acting on the nodes on the RBC surface. A lattice-Boltzmann method was combined with this discrete model to simulate 200 densely packed RBCs in three dimensional flow.

Among these methodologies and models mentioned above, we want to combine the immersed boundary method with spring model since we intend to simulate the mixture of deformable and rigid particles in microvessels in near future. We have already developed very efficient methodologies, called