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DNS OF FORCED MIXING LAYER

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Abstract. The non-dimensional form of Navier-Stokes equations for two dimensional mixing layer flow are solved using direct numerical simulation. The governing equations are discretized in streamwise and cross stream direction using a sixth order compact finite difference scheme and a mapped compact finite difference method, respectively. A tangent mapping of $y = \beta \tan(\pi\zeta/2)$ is used to relate the physical domain of y to the computational domain of ζ. The third order Runge-Kutta method is used for the time-advancement purpose. The convective outflow boundary condition is employed to create a non-reflective type boundary condition at the outlet. An inviscid (Stuart flow) and a completely viscous solution of the Navier-Stokes equations are used for verification of the numerical simulation. The numerical results show a very good accuracy and agreement with the exact solution of the Navier-Stokes equation. The results of mixing layer simulation also indicate that the time traces of the velocity components are periodic. Results in self-similar coordinate were also investigated which indicate that the time-averaged statistics for velocity, vorticity, turbulence intensities and Reynolds stress distribution tend to collapse on top of each other at the flow downstream locations.

Key Words. Mixing Layer, Compact Finite Difference, Mapped Finite Difference, Self-Similarity.

1. Introduction

The plane mixing layer is characterized by the merging of two co-flowing fluid streams with different velocities. Typically, the two streams are separated by an impermeable object upstream of the confluence of these streams. This situation is illustrated in Fig1 [1]. Downstream of the confluence, the two streams exchange momentum as they come into intimate contact with each other. The mixing layer itself is defined by the region in which this merging process is occurring. Being such a simple configuration, it stands to reason that the mixing layer is one of the more common flows experienced in nature. Mixing layer are encountered in many application such as combustion furnaces, chemical lasers, the lip of an intake valve in an internal combustion engine and the trailing edge of a turbine blade.
Schlichting [2] shows that the boundary layer equations are valid for mixing layer at high Reynolds number. He assumed that two initially unperturbed parallel flow streams with velocities U, λU interact as a consequence of friction with one another from the position $x = 0$ to downstream. For the low values of viscosity ν, the transition from the velocity U to the velocity λU takes place in a thin mixing zone, in which the transverse component v of the velocity is small in compare with the longitudinal velocity component u. This boundary layer equation without the

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presence of pressure gradient term is valid. Since there is no characteristic length in this problem, similar solutions are found.
Perhaps the first major experimental investigation of the mixing layer was undertaken by Liepmann and Laufer [3]. Many of the statistical quantities were explored in this investigation. Brown and Roshko [4] performed a study on density effects in the mixing layer which would lay the groundwork for a virtual revolution in turbulence. Mungal and Dimotakis [5] considered the mixing and combustion of two reactants in a gaseous mixing layer.
With advent of large scale computers, there has been a veritable explosion of numerical work done in concert with the experiments in an effort to understand the physics of mixing layer. Generally, there are two advanced methods of computing turbulent flow: large eddy simulation (LES) and direct numerical simulation (DNS).
In LES, a low-pass spatial filtering is applied to the Navier-Stokes equation and the filtered equations are solved directly. It is most promising for engineering flow of low-to-moderate Reynolds numbers [6], [7]. A review of LES for incompressible flow can be found in [8].
The main purpose of DNS is to solve (to best of our ability) for the turbulent velocity field without the use of turbulent modeling. This condition means that the Navier-Stokes momentum equation for fluid must be solved exactly, which is not a simple task [9], [10]. Thus, any DNS code is very time consuming and the extensive storage requirements. The DNS requires a large number of grid points and time steps to reach a statistically steady state and are usually limited to relatively low Reynolds numbers. With the advantage of powerful super computers, numerical simulation have become a viable tool for investigating mixing layer flows such as high speed mixing layer [11], particle laden mixing layer [12] and mixing layer with chemical reaction [13], [14].
In this paper the governing equations are derived from the full incompressible Navier-Stocks equations. These are solved in a domain which is finite in the streamwise direction, x and doubly infinite in the cross stream direction of y. In the x direction a high order compact finite difference scheme is used. In the y direction, a mapped compact finite difference method is employed. All quantities are non-dimensionalized by the appropriate characteristic scales of the nixing layer flow. Specially, all lengths are normalized by the vorticity thickness of the reference velocity profile, δ_{ω_0}. Velocities are normalized by free streams streamwise velocity difference ΔU, where $\Delta U = U_1 - U_2$ and those pertaining to time are normalized by $\delta_{\omega_0}/\Delta U$.
The mean component of the streamwise velocity at the inlet plane of the domain, termed "reference" velocity also, is represented by:

$$U_0\left(y\right) = \frac{1}{2}\left\{\left({}^{1}\!/\!_{\lambda}\right) + \tanh\left(2y\right)\right\}. \tag{1}$$

where the aforementioned normalization has been incorporated. The parameter $\lambda = \left[U_1 - U_2\right] / \left[U_1 + U_2\right]$ represents a measure for the intensity of shearing of the layer. The value of 2 in the argument for the tanh is required to make the vorticity thickness of this profile consistent with the implicit normalization of z on δ_{ω_0}.

2. The Governing Equations

Figure (1) shows the coordinate system and the computational domain in which the governing equations for the incompressible mixing layer flow are solved. The