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ASSOCIATING A LIMIT PERTURBATION MODEL IN 3D WITH REDUCED MESHES FOR SIMULATIONS OF THE LOCALIZATION OF CERTAIN ELECTROMAGNETIC INHOMOGENEITIES

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Abstract. Simulations of the localization of certain small electromagnetic inhomogeneities, in a three-dimensional bounded domain, are performed by making use of a framework recently introduced by the author and of a non-standard discretization process of this domain. This framework is based on a limit model in electric field and on the combination of a limit perturbation model in the tangential boundary trace of the curl of the electric field, with a Current Projection method or an Inverse Fourier method. As opposed to our recent paper relating to this framework and to experiments requiring the usual discretization process of the domain, inhomogeneities that are one order of magnitude smaller are numerically localized here.

Key Words. inverse problems, Maxwell equations, electric fields, inhomogeneities, Electrical Impedance Tomography, Current Projection method, FFT, numerical boundary measurements, edge elements, least square systems, Incomplete Modified Gram-Schmidt preconditioning, composite numerical integrations.

1. Introduction

Several recent works, related to Electrical Impedance Tomography, deal with the localization of inhomogeneities that are of small diameters. These works (see e.g. [1, 2, 3, 4, 5, 10, 12, 18]) present tools and numerical methods for solving the localization problem in diverse settings (conductivity inhomogeneities, elastic inhomogeneities, ...). For the localization of a finite number of small electromagnetic inhomogeneities contained in a three-dimensional bounded domain, from a finite number of boundary measurements, H. Ammari, M. Vogelius & D. Volkov propose in [6] a practical tool. The inverse problem underlying the localization is based in [6] on the time-harmonic Maxwell equations, and the proposed tool is an asymptotic formula for perturbations in the electromagnetic fields, due to the presence of such inhomogeneities. It allows one in particular to evaluate boundary measurements of "voltage" type that are then used as data of the inversion algorithm — aimed at locating the inhomogeneities. This tool has been recently considered by M. Asch & S.M. Mefire [8, 9] for numerically performing the localization of such inhomogeneities in various experimental contexts (consideration of diverse frequencies, consideration of inhomogeneities of different smallness, experiments with diverse inversion algorithms).

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In the numerical investigations of [8], where the time-harmonic Maxwell equations and the formula for perturbations are considered in electric field, it is however observed that such inhomogeneities cannot be localized from this formula in the context of very low frequencies. This observation led to an essential question, that of knowing whether these inhomogeneities can be localized from the limit model of equations and the limit perturbation model obtained by letting the frequency vanish in the time-harmonic Maxwell equations in electric field and in the formula for perturbations in the tangential trace of the curl of the electric field allowing one to evaluate boundary measurements. This question has been recently answered by the author in [15]. The numerical investigations performed in [15] indicate that these limit models lead to the localization of inhomogeneities that are not purely electric. However, the inhomogeneities considered in [15] are not sufficiently small in such a way that we can assimilate the geometric configurations of numerical experiments of [15] to concrete configurations from a physical point of view. Typically, these experiments required, in particular, for the numerical evaluation of measurements, a finite element method based on "full" conforming meshes of the domain whereas, when the domain contains very small inhomogeneities, such meshes (that take into account implicitly the conforming discretization of each imperfection) prohibit simulations of the localization as far as memory storage and CPU time are concerned. In fact, in presence of such inhomogeneities, such a mesh, deriving from the usual triangulation process of the domain, leads to a too large number of unknowns of the discrete system (associated with the discrete formulation in electric field) that must be solved for each evaluation of measurement; especially as the domain is three-dimensional and as mixed finite elements are used.

As opposed to [15], where we were limited in numerical investigations by the smallness of the inhomogeneities, configurations of much smaller inhomogeneities will be treated here.

In this work, the simulations of the localization will be based on the aforementioned limit perturbation model, and on finite element meshes called, as in [9], the *reduced meshes*. Such a mesh of the domain, aimed at overcoming the drawbacks inherent in full meshes, represents a conforming mesh whose size is bigger than the largest of the diameters of the inhomogeneities present in the domain, and is (explicitly) combined with integration meshes for taking into account the characteristics of these small inhomogeneities.

This work is subdivided into five sections. In Section 2, we introduce from [15] the limit model in electric field and the limit perturbation model that allows us, in particular, to evaluate boundary measurements. We present in Section 3 the (direct) computation of the electric field required in the evaluation of each measurement. Typically, this computation is based on a discrete formulation resulting from the combination of a reduced mesh, with Nédélec's edge elements and nodal finite elements. As this formulation provides a rectangular matrix system, we are concerned with a least squares approach for solving the system and therefore determining the discrete electric field. Section 4 deals with extensive simulations, making then use of reduced meshes, and considering two localization procedures: the one based on a Current Projection method (for the single inhomogeneity context) and the one deriving from an Inverse Fourier method (for the multiple inhomogeneities context). We describe localization results obtained, in various contexts, with each one of these procedures and also compare some results in the single inhomogeneity context. Some conclusions are reported in Section 5.