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NUMERICAL IDENTIFICATION OF MAGNETIC PERMEABILITY

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Abstract. This work is concerned with the analysis on a numerical reconstruction of the magnetic permeability. The ill-posed problem is solved through a stabilized nonlinear minimization system by an appropriately selected Tikhonov regularization. The existence and stability of the optimization system are demonstrated. The nonlinear optimization problem is approximated by an edge element method, whose convergence is established.

Key Words. Numerical identification, Maxwell system, permeability, edge element method, stability, convergence.

1. Introduction

In this work we are interested in the numerical reconstruction of the distribution of the magnetic permeability in the following Maxwell system:

$$\varepsilon(\mathbf{x})\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{H} = \mathbf{J} \text{ in } \Omega\,, \tag{1.1}$$

$$\nu(\mathbf{x})\frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} = 0 \text{ in } \Omega\,, \tag{1.2}$$

where $\mathbf{E}$ and $\mathbf{H}$ represent the electric and magnetic fields of the physical medium which occupies a domain Ω in $\mathbf{R}^3$. The Maxwell system (1.1)-(1.2) is formed by the Ampere's law (1.1), and Faraday's law (1.2), and plays an important role in most applications that involve electromagnetism. The coefficients $\varepsilon(\mathbf{x})$ and $\nu(\mathbf{x})$ in (1.1) and (1.2) are the electric permittivity and magnetic permeability of the medium in Ω, while $\mathbf{J}$ is the applied electric current density. When the physical properties of the medium involved are known, i.e. $\varepsilon(\mathbf{x})$ and $\nu(\mathbf{x})$ are given, one can solve the system (1.1)-(1.2) to find the behaviors of the electric and magnetic field $\mathbf{E}$ and $\mathbf{H}$ in Ω. This is usually called a direct Maxwell problem. While in many applications, the inverse Maxwell problem may be more interesting and practically important, where the electric or magnetic property of the physical medium occupied by Ω is unknown. But knowing them is indispensable to some research investigations in Ω or to a good understanding of the physical medium Ω and how the fields $\mathbf{E}$ and $\mathbf{H}$ behave in Ω. In this work we shall consider the case when the electric permittivity of the physical medium occupied by Ω is available, but the magnetic permeability of the medium is unknown. In order to recover the magnetic permeability of the medium, we need to have some extra measurement data from the electric field $\mathbf{E}$

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or magnetic field $\mathbf{H}$. We shall assume the measurement data of $\mathbf{E}$ is available in some small subregion inside Ω. So the inverse problem to be considered can be formulated as follows:

Inverse Problem I. Let ω be a subregion in Ω. Given the measurement data

$$\mathbf{E}^{\delta}(\mathbf{x},t) \approx \mathbf{E}(\mathbf{x},t)\,, \quad (\mathbf{x},t) \in \omega \times (0,T)\,, \tag{1.3}$$

we will reconstruct the distribution profile of the magnetic permeability $\nu(\mathbf{x})$ in the entire domain Ω.

Noting that only the measurement data of the electric field $\mathbf{E}$ is available in Inverse Problem I, but the Maxwell system (1.1)-(1.2) involves both the electric and magnetic fields. So it is more natural to deal with a system that involves only the electric field. To do so, taking the time derivative on both sides of equation (1.1) and applying equation (1.2) gives the following electric field equation

$$\varepsilon(\mathbf{x})\frac{\partial^2 \mathbf{E}}{\partial t^2} + \nabla \times (\nu^{-1}(\mathbf{x})\nabla \times \mathbf{E}) = \frac{\partial \mathbf{J}}{\partial t} \quad \text{in} \quad \Omega\,. \tag{1.4}$$

Complementary to this electric field equation we shall consider the boundary condition

$$\mathbf{E} \times \mathbf{n} = 0 \quad \text{on} \quad \partial\Omega \tag{1.5}$$

and the initial conditions

$$\mathbf{E}(\mathbf{x},0) = \mathbf{E}_0(\mathbf{x})\,, \quad \mathbf{E}_t(\mathbf{x},0) = \mathbf{E}_1(\mathbf{x}) \quad \text{in} \quad \Omega\,. \tag{1.6}$$

For the known electric permittivity $\varepsilon(\mathbf{x})$, we know physically that it should be always bounded below and above. Hence we will assume that

$$\varepsilon_0 \le \varepsilon(\mathbf{x}) \le \varepsilon_1 \quad a.e. \quad \text{in} \quad \Omega\,, \tag{1.7}$$

where ε_0 and ε_1 are two positive constants.

Inverse problems of parameter identifications have attracted a great attention in the recent two decades due to their practically important applications in engineering and scientific computing; see, e.g. [1] [5] and the references therein. The mathematical and numerical analysis of identifications of parameters in many partial differential equations were available in the literature, see [1] [2] for the elliptic system; and [5] [6] [8] [11] for parabolic systems. But very little has been done for the analysis of numerical reconstruction of the parameters in the electromagnetic Maxwell system. This motivates the central topic of this current investigation.

2. Problem formulation and existence of solutions

In this section we shall formulate the ill-posed Inverse Problem I stated in Section 1 as a stabilized minimization system and establish the existence of the solutions and stability of the minimization formulation. For the sake of convenience, we shall rewrite the electric system (1.4) as

$$\varepsilon(\mathbf{x})\frac{\partial^2 \mathbf{E}}{\partial t^2} + \nabla \times (\mu(\mathbf{x})\nabla \times \mathbf{E}) = \mathbf{j} \quad \text{in} \quad \Omega \tag{2.1}$$

where $\mathbf{j} = \frac{\partial \mathbf{J}}{\partial t}$, and $\mu(\mathbf{x}) = \nu^{-1}(\mathbf{x})$ is the magnetic susceptibility. If $\mu(\mathbf{x})$ is known, then the magnetic permeability $\nu(\mathbf{x})$ targeted in Inverse Problem I can be obtained by taking the simple inverse of $\mu(\mathbf{x})$. So in the subsequent sections, we shall address