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A FAST ALGORITHM FOR VECTORIAL TV-BASED IMAGE RESTORATION

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Abstract. In this paper, we first extend a simple algorithm proposed by Jia et al. [16] to color/vectorial images, and then apply the vectorial algorithm to some variational models for image restoration problems including color image denoisings with the red-green-blue (RGB) and chromaticity-brightness (CB) color representations, CB based colorization and image inpainting. The variational models are all total variation (TV)-based. The proposed vectorial algorithm is simple and straightforward to implement. Some numerical experiments show that it is fast and efficient.

Key Words. Image restoration, vectorial TV model, split Bregman iteration, color denoising, CB based colorization, TV-based inpainting.

1. Introduction

Image restoration is an important research field in image processing. It is often considered as a pre-processing step for other image tasks such as image segmentation, image registration and so on. Image restoration includes many aspects, for example denoising, deblurring, inpainting, colorization, etc.

Over the past twenty years, total variation(TV)-based models proposed firstly by Rudin, Osher, and Fatemi in [23] for gray image denoising have become very popular. They have had very good applications in image denoising [1, 9, 8, 21], deblurring [12, 15], inpainting [10, 19, 11, 24], colorization [18], and so forth. There have been a lot of methods to solve these TV-based models like standard regularized approach [23, 1], primal-dual method [7], duality based method [5], split Bregman method [14], recent augmented Lagrangian method [25, 26, 27], etc. The classical algorithms (standard regularized approach or explicit gradient descent flow) often need to solve discrete Euler-Lagrange equations [23, 1, 2, 13], whose computational speed is very slow due to the regularization process of the TV-norm. Later, Chambolle [5] proposed a fast algorithm based on the dual formulation of TV-norm, which avoided the regularization of TV-norm and hence speeded up the computation dramatically. Recently, Goldstein and Osher [14] gave a novel algorithm called "split Bregman" method to solve these TV-based models. The key of their method is that they de-coupled the ℓ_1 and ℓ_2 portions of TV model and transformed the ℓ_1 regularized term to compressed sensing (CS) problems, which can be fast solved by the Bregman iteration and shrinkage. The convergence of the split Bregman iteration was shown in [14, 17] under the assumption that the resulting subproblem is solved exactly. Cai et al. [4] had also proven that the alternating split Bregman iterations are convergent when the number of inner iterations is fixed to be one.

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However, these iteration schemes [14, 17, 4] still require solving a partial differential equation in each iteration step. The augmented Lagrangian method [25, 26, 27] was presented to solve TV models via a splitting technique and the Lagrange multiplier method. Some subproblems can be efficiently solved by shrinkage and fast Fourier transformation (FFT) implementation, where the FFT technique is used for solving a differential equation, and thus cuts down the computational time. But it is still less efficient than the direct closed form solution. More recently, Jia and Zhao [16] proposed a fast and simple algorithm to solve the Rudin-Osher-Fatemi (ROF) model/ TV denoising model. Their algorithm did not include any partial differential equations and had very simple iteration steps, which saved more computational time. What's more, they also gave a rigorous proof of the convergence of their algorithm.

In this paper, we extend Jia and Zhao's algorithm [16] to vectorial TV model, and then apply it to vector-valued image restoration problems such as a color image restoration. Here, we mainly focus on three restoration problems: color denoisings by vectorial TV-based denoising models in the red-green-blue(RGB) and chromaticity-brightness(CB) representations of color images [1, 9, 8]; image colorization based on CB color model [18]; and image inpainting [10, 19] by TV-based model for gray and color images. The proposed algorithm has several advantages. First, it has a very simple form, which will be favorable to making code. Then, the number of iterations to reach the solution is low, which gives a fast algorithm. Finally, the algorithm converges to the solution of the original vectorial TV minimization problem if appropriate parameters are chosen.

This paper is organized as follows. In Section 2, we introduce some notations and extend Jia and Zhao's algorithm to vector-valued functions so that the speed of the vectorial image processing is faster. The applications of the algorithm to image restoration including color image denoisings based on RGB and CB color representations, CB-based colorization and inpainting for gray and color images are shown in Section 3. At last, in Section 4, we present a brief conclusion.

2. Proposed algorithm for vectorial TV minimization

2.1. Notations. As in [17], we adopt the discrete form of the vectorial TV model. Let us consider a q-dimensional/channel image $\mathbf{u}$ defined on a rectangular domain Ω as follows:

$$\begin{aligned} \mathbf{u}:\Omega &\rightarrow \mathbb{R}^q, \\ (x,y) &\rightarrow \mathbf{u}(x,y)=\big(u_1(x,y),u_2(x,y),\cdots,u_q(x,y)\big). \end{aligned}$$

Discretizing the image domain Ω to some grid points, then

$$\begin{aligned} \mathbf{u}:\{1,\cdots,M\}\times\{1,\cdots,N\} &\rightarrow \mathbb{R}^q, \\ (m,n) &\rightarrow \big(u_1(m,n),u_2(m,n),\cdots,u_q(m,n)\big), \end{aligned}$$

where $M,N\geq 2$ and $q\geq 1$.

When $q=1$, the image is scalar; otherwise, the image is vector-valued.

We shall use the following norm and inner product notations:

$$\|\mathbf{u}\|_p := \Bigg(\sum_{\substack{1\le m\le M\\ 1\le n\le N}} |\mathbf{u}(m,n)|^p\Bigg)^{1/p}, \quad \text{for} \quad 1\le p<\infty,$$

$$\big\langle \mathbf{u}(m,n),\mathbf{v}(m,n)\big\rangle := \sum_{i=1}^{q} u_i(m,n)\cdot v_i(m,n),$$