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DOMAIN DECOMPOSITION METHODS WITH GRAPH CUTS ALGORITHMS FOR IMAGE SEGMENTATION

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Abstract. Recently, it is shown that graph cuts algorithms can be used to solve some variational image restoration problems, especially connected with noise removal and segmentation. For very large size images, the usage for memory and computation increases dramatically. We propose a domain decomposition method with graph cuts algorithms. We show that the new approach costs effective both for memory and computation. Experiments with large size 2D and 3D data are supplied to show the efficiency of the algorithms.

Key words. Multiphase Mumford-Shah, Graph cuts, Image segmentation, Domain decomposition

1. Introduction

Segmentation is one of the fundamental problems in image processing and computer vision tasks. The result of image segmentation is a set of contours extracted from the image, or a set of regions that collectively cover the entire image. Mumford and Shah model [26] is an effective tool for region based image segmentation. This model is robust to noise and can segment objects without edges. However, the minimization problem is difficult to solve numerically.

The level set method [13, 27] was first introduced to solve the Mumford-Shah functional by Chan and Vese in [7, 33]. Chan and Vese model achieves great success in image segmentation due to its advantages in obtaining large convergence range and handling the topological changes. A lot further works of Chan and Vese model were done in [23, 28]. Some variants of the level set method, so-called "Piecewise Constant Level Set Method" (PCLSM), were proposed in [24, 25, 29]. This PCLSM can identify several interfaces by one single level set function, which makes it easier to solve the Mumford-Shah model.

Traditionally, methods based on gradient descent are often used for solving the Mumford-Shah models, see [24, 25, 29]. These methods are normally slow and difficult to find global minimizers. Recently, many works have been done on applying graph cuts algorithms for image segmentation [5, 3, 21, 12, 34]. They are proven to be more efficient for solving this kind of energy minimization problems. The connection between graph cuts and variational problems has been established in [2, 18, 10, 4]. For Mumford-Shah segmentation, some work using graph cuts optimization for two-phase model has been done in [9] and [14]. For multiphase Mumford-Shah model, the methods of [5, 22, 18] can be adopted for solving the corresponding energy problems. In this work, we shall follow the approach given in [1]. In [1], the authors used the level set formulation of Mumford-Shah model [27]

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and adopted the graph construction method in [18, 19] to multiphase Mumford-Shah model. However, when the images become large and the number of phases increases, especially for 3D segmentation cases, both computational cost and memory usage increase greatly. In this work we try to find some remedies for these difficulties and show that we could get some algorithms which have quite high efficiency as well as low memory usage. We propose a method combining the domain decomposition method with graph cuts algorithms.

The paper is organized as follows. In section 2, we review the PCLSM and its applications to the Mumford-Shah model. In section 3, we review the graph cuts algorithm of [1] to the multiphase Mumford-Shah model. In Section 4, we combine the domain decomposition methods with the graph cuts idea to solve the Mumford-Shah model. Some implementation details are supplied in Section 5. Finally, in Section 6, we carry out some experiments by our algorithms and compare the results with the original graph cuts algorithm.

2. Mumford-Shah model with PCLSM

2.1. Mumford-Shah model. The Mumford-Shah model is a well known model for image segmentation problem [26]. In the model, Ω is a bounded domain and $u^0(x)$ is the input image. We search for n interfaces Γ_i and an approximation image u by minimizing the following energy functional

$$E(u,\Gamma_i) = \int_\Omega (u-u^0)^2 dx + \mu \int_{\Omega \setminus \bigcup_i \Gamma_i} |\nabla u|^2 dx + \sum_{i=1}^{n} \gamma \int_{\Gamma_i} ds. \tag{1}$$

where μ and γ are nonnegative constants and $\int_{\Gamma_i} ds$ is the length of the boundary of interfaces Γ_i. The most popular way to solve this minimization problem is applying the level set method [7], especially the piecewise constant level set Mumford-Shah model. For such case, the second term vanishes in the minimization functional.

2.2. Piecewise constant level set method. In [24, 25, 29], the piecewise constant level set method (PCLSM) was proposed and applied to the Mumford-Shah model. The main idea of PCLSM is to seek a partition of the domain Ω into n subdomains Ω_i, $i = 1, 2, \cdots, n$. The essential idea is to use a piecewise constant level set function ϕ to identify the subdomains

$$\phi = i \quad in \quad \Omega_i. \tag{2}$$

Once the function ϕ is identified, we can construct the corresponding characteristic functions for each subdomain Ω_i as

$$\psi_i = \frac{1}{\alpha_i} \prod_{j=1, j\neq i}^{n} (\phi - j), \quad \text{with} \quad \alpha_i = \prod_{k=1,k\neq i}^{n} (i-k). \tag{3}$$

If ϕ is defined as in (2), we have $\psi_i(x) = 1$ for $x \in \Omega_i$, otherwise we have $\psi_i(x) = 0$. Based on these characteristic functions, we can extract the geometrical information of the boundaries of the subdomains $\{\Omega_i\}_{i=1}^n$. For example, the length of the interfaces surrounding each subdomain Ω_i, $i = 1, 2, \cdots, n$, should be

$$Length(\partial\Omega_i) = \int_\Omega |\nabla(\psi_i)| dx. \tag{4}$$

For some given values $c_i, i = 1, 2, \cdots n$, define

$$u = \sum_{i=1}^{n} c_i \psi_i. \tag{5}$$