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AN ELASTO-VISCOPLASTIC CONTACT PROBLEM: AN A POSTERIORI ERROR ANALYSIS AND COMPUTATIONAL EXPERIMENTS

JOSÉ R. FERNÁNDEZ

Abstract. In this paper, we reconsider a contact problem between an elasto-viscoplastic body and a deformable obstacle. The contact is modeled by the classical normal compliance contact condition. Then, fully discrete approximations are obtained by using the finite element method to approximate the spatial variable and the forward Euler scheme to discretize time derivatives. An a posteriori error analysis is provided and upper and lower error bounds are obtained. Finally, some two-dimensional numerical simulations are presented to demonstrate the accurary and the behavior of the error estimators.

Key words. Elasto-viscoplasticity, normal compliance contact, fully discrete approximations, a posteriori error estimates, finite elements, numerical simulations.

1. Introduction

During the past twenty years many problems have been studied dealing with elasto-viscoplastic materials modeled using the constitutive law introduced in [9] (see the monograph [19] and its references). Then, numerous nonlinear problems including this kind of materials (as, for instance, contact problems) were considered (see, e.g., [1, 2, 5, 6, 10, 13, 16, 24, 25, 26], the well-written monograph [17] and the large number of references cited therein). We note that, as it was justified in [9], this law is mechanically correct and it can be used for the modeling of some types of metals or rocks since it allows both creep and relaxation phenomena.

In this work, we revisite the contact problem between an elasto-viscoplastic body and a deformable obstacle. The contact is modeled using the classical normal compliance contact law described, for example, in [20, 21]. This problem was already studied in [14] (see also the paper [11] where internal variables were also considered). A priori error estimates were proved there (see Section 3 where they are recalled) and numerical simulations were provided in order to show the accuracy of the algorithm and the behavior of the solution. However, even if many other papers were published since then, only a priori error estimates were obtained. Recently, an a posteriori error analysis was presented in [12] in the case without contact, extending some arguments already applied in the study of the heat equation (see, e.g., [22, 23, 28]), some parabolic equations ([3]) or the Stokes equation ([4]). Hence, this work continues the above referenced work by Fernández and Hild [12], extending the analysis presented there to the case including the contact with a deformable obstacle and also the previous paper [14], where the a priori error analysis was conducted. Moreover, here we also perform several two-dimensional numerical simulations in order to demonstrate the accuracy of the algorithm and the behavior of the error estimators.

The paper is outlined as follows. In Section 2 the mechanical model and its variational formulation are briefly described following the notation and assumptions

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introduced in [14]. Then, fully discrete approximations are provided in Section 3, by using the finite element method to approximate the spatial variable and the forward Euler scheme to discretize the time derivatives. An a priori error analysis obtained in [14] is recalled. Then, by using some results obtained in the study of the heat equation, an a posteriori error analysis is done in Section 4, providing an upper bound for the error, Theorem 4.1, and a lower bound, Theorem 4.2. Finally, some two-dimensional numerical simulations are presented in Section 5 in order to demonstrate the accuracy and the behavior of the error estimators introduced in the previous section.

2. Mechanical and variational formulations

In this section, we present a brief description of the contact problem between an elasto-viscoplastic body and a deformable obstacle (further details can be found in [14, 17]).

Denote by $\mathbb{S}^d$ the space of second order symmetric tensors on $\mathbb{R}^d$ and by "$\cdot$" and $|\cdot|$ the inner product and the Euclidean norms on $\mathbb{R}^d$ and $\mathbb{S}^d$.

Let $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, denote a domain occupied by an elasto-viscoplastic body with a smooth boundary $\Gamma = \partial\Omega$ decomposed into three disjoint parts Γ_D, Γ_F and Γ_C such that meas $(\Gamma_D) > 0$ and meas $(\Gamma_C) > 0$. Moreover, let $[0, T]$, $T > 0$, be the time interval of interest and denote by $\boldsymbol{\nu}$ the unit outer normal vector to Γ. The body is being acted upon by a volume force of density $\boldsymbol{f}_0$, it is clamped on Γ_D and surface tractions with density $\boldsymbol{f}_F$ are applied on Γ_F. Finally, we assume that the body may come in contact with a deformable obstacle, on the boundary part Γ_C, which is located at a distance g measured along the outward unit normal vector $\boldsymbol{\nu}$ (see FIGURE 1).

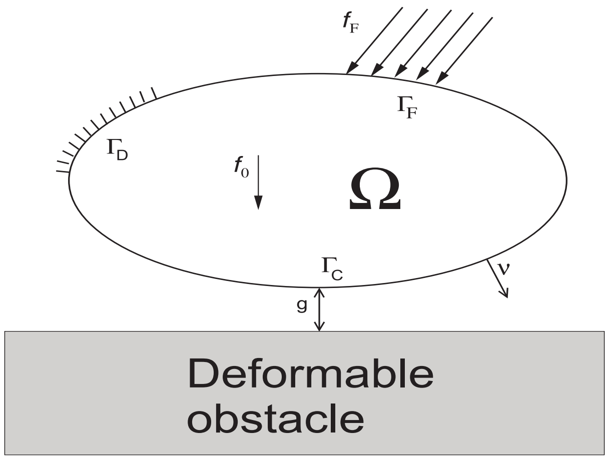

FIGURE 1. Physical setting: an elasto-viscoplastic body in contact with a deformable obstacle.

Let $\boldsymbol{x} \in \Omega$ and $t \in [0, T]$ be the spatial and time variables, respectively, and, in order to simplify the writing, we do not indicate the dependence of the functions on $\boldsymbol{x}$ and t. Moreover, a dot above a variable represents the derivative with respect to the time variable.

Let us denote by $\boldsymbol{u} = (u_i)_{i=1}^d$, $\boldsymbol{\sigma} = (\sigma_{ij})_{i,j=1}^d$ and $\boldsymbol{\varepsilon}(\boldsymbol{u}) = (\varepsilon_{ij}(\boldsymbol{u}))_{i,j=1}^d$ the displacement field, the stress tensor and the linearized strain tensor, respectively.