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ADAPTIVE MESH REFINEMENT FOR ELLIPTIC INTERFACE PROBLEMS USING THE NON-CONFORMING IMMERSED FINITE ELEMENT METHOD

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Abstract. In this paper, an adaptive mesh refinement technique is developed and analyzed for the non-conforming immersed finite element (IFE) method proposed in [27]. The IFE method was developed for solving the second order elliptic boundary value problem with interfaces across which the coefficient may be discontinuous. The IFE method was based on a triangulation that does not need to fit the interface. One of the key ideas of IFE method is to modify the basis functions so that the natural jump conditions are satisfied across the interface. The IFE method has shown to be order of $O(h^2)$ and $O(h)$ in L^2 norm and H^1 norm, respectively. In order to develop the adaptive mesh refinement technique, additional priori and posterior error estimations are derived in this paper. Our new a-priori error estimation shows that the generic constant is only linearly proportional to ratio of the diffusion coefficient β^- and β^+, which improves the corresponding result in [27]. We also show that a-posteriori error estimate similar to the one obtained by Bernardi and Verfürth [4] holds for the IFE solutions. Numerical examples support our theoretical results and show that the adaptive mesh refinement strategy is effective for the IFE approximation.

Key Words. interface, immersed finite element, adaptive mesh

1. Introduction

The main purpose of this paper is to develop adaptive mesh refinement techniques for the immersed finite element (IFE) method proposed in [27]. Along this line, we also discuss a-priori and a-posteriori error estimates for the immersed finite element method. The IFE method was developed for the following interface problem:

$$-\nabla \cdot (\beta \nabla u) = f, \ (x,y) \in \Omega \qquad (1)$$
$$u \mid_{\partial\Omega} = g,$$

together with the natural jump conditions across the interface $\tilde{\Gamma}$:

$$[u] \mid_{\tilde{\Gamma}} = 0, \qquad (2)$$
$$[\beta u_n] \mid_{\tilde{\Gamma}} = 0. \qquad (3)$$

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Here, $\Omega \subset \mathbb{R}^2$ is a convex polygonal domain, the interface $\tilde{\Gamma}$ is a curve separating Ω into two sub-domains Ω^-, Ω^+ such that $\Omega = \Omega^- \cup \Omega^+ \cup \tilde{\Gamma}$, and the coefficient $\beta(x, y)$ is a piecewise continuous function

$$\beta(x,y) = \begin{cases} \beta^-(x,y), & (x,y) \in \Omega^-, \\ \beta^+(x,y), & (x,y) \in \Omega^+, \end{cases}$$

see the diagram in Fig. 1 for an illustration.

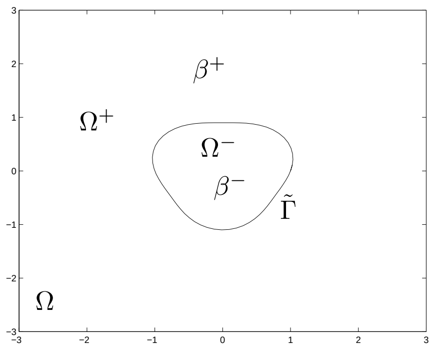

FIGURE 1. A rectangular domain $\Omega = \Omega^+ \cup \Omega^-$ with an immersed interface $\tilde{\Gamma}$. The coefficients $\beta(\mathbf{x})$ may have a jump across the interface.

The interface problem considered here appears in many engineering and science applications. The immersed finite element (IFE) space was first introduced in [27], in which some preliminary analysis and numerical results are reported. The new IFE method has been developed for non-homogeneous jump conditions (with nonzero right hands of (2) and (3)) in [25]. Some related work can be found in [19, 20, 24, 28].

The basic idea of the immersed finite elements is to form a partition $\Im_h$ independent of interface $\tilde{\Gamma}$ so that partitions with simple regular structures can be used to solve an interface problem with a rather complicated or varying interface. Obviously, triangles in a partition can be separated into two classes:

• Non-interface triangles: The interface $\tilde{\Gamma}$ either does not intersect with this triangle, or it intersects with this triangle but does not separate its interior into two nontrivial subsets.

• Interface triangles: The interface $\tilde{\Gamma}$ cuts through its interior.

In a non-interface triangle, the standard linear polynomials is employed as local basis functions. However, in an interface triangle, a piecewise linear polynomial is defined in the two subsets formed by the interface in a way that the functions satisfy the natural jump conditions (either exactly or approximately) on the interface and retain specified values at the vertices of the interface triangle. The immersed finite element space defined over the whole domain Ω can then be constructed through the standard finite element assembling procedure. We refer the readers to [9–12, 15, 18, 23, 26] for more background materials about immersed interface and immersed finite element methods as well as their applications.

Without loss of generality, we assume that the triangles in the partition have the following features: